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# On the relative pricing of long maturity S&P 500 index options and CDX tranches

### Pierre Collin-Dufresne Robert Goldstein Fan Yang

May 2010

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# Securitized Credit Markets Crisis

- Pre-crisis saw large growth in securitized credit markets (CDO).
- Pooling and tranching used to create 'virtually risk-free' AAA securities, in response to high demand for highly rated securities.
- > During the crisis all AAA markets were hit hard:
  - Home equity loan CDO prices fell (ABX.HE AAA < 60%).</li>
  - Super Senior (30-100) tranche spreads > 100bps.
  - CMBX.AAA (super duper) >750bps.
- Raises several questions:
  - Q? Were ratings incorrect (ex-ante default probability higher than expected)?
  - Q? Are ratings sufficient statistics (risk  $\neq$  expected loss)?
  - Q? Were AAA tranches mis-priced (relative to option prices)?
- Many other surprises:
  - Corporate Credit spreads widened (CDX-IG > 200bps).
  - Cash-CDS basis negative (-200 bps for IG; -700bps for HY).
  - LIBOR-Treasury and LIBOR-OIS widened (> 400bps).
  - Long term Swap spreads became negative (30 year swap over Treasury < -50 bps).
  - Defaults on the rise (Bear Stearns, Lehman).

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### Evidence from ABX markets

► ABX.HE (subprime) AAA and BBB spreads widened dramatically (prices dropped)

J.P.Morgan DataQuery



J.P.Morgan Inc.

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### Evidence from CMBX markets

> CMBX (commercial real estate) AAA spreads widened even more dramatically

# J.P.Morgan DataQuery

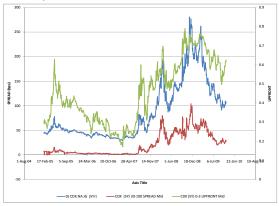


J.P.Morgan Inc

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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### Corporate IG CDX Tranche spreads

The impact on tranche prices was dramatic

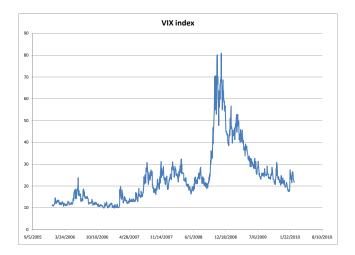


- Implied correlation on equity tranche hit > 40%
- Correlation on Super-Senior tranches > 100%(!) with standard recovery assumption
- Relative importance of expected loss in senior tranche versus in equity tranche indicates increased crash risk.

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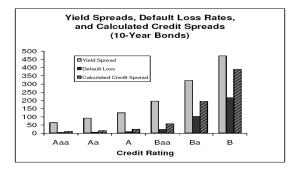
# Evidence from S&P500 Option markets

> Implied volatility index widened dramatically: increased market and crash risk.



Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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The Credit spread puzzle (pre-crisis)



source: Huang and Huang (2003)

- Huang and Huang (2003) find that Structural models, when calibrated to match average loss rate, tend to underpredict yield spreads
- Chen, Collin-Dufresne, Goldstein (2008) find that standard models cannot explain the level of observed spreads because:
  - (i) historical expected loss rates have been low, and
  - (ii) Idiosyncratic risk on typical IG bonds is very high ( $\sim 3/4$  of total risk).

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# CDO collateral typically have high beta due to diversification

- ► Coval, Jurek, Stafford propose theory for large growth in structured product markets:
  - Posit that ratings are sufficient statistic for expected loss.
  - Tranching process pools risky securities (e.g., BBB) to create lower risk (e.g., AAA) and higher risk (e.g., Z) securities by creating different levels of subordination (tranches).
  - By nature of that process senior tranches have more systematic risk and therefore should have higher expected return for given expected loss (~ rating).
  - However investors focus only on expected loss (~ rating).
- ⇒ Effectively, according to CJS, the banking sector exploits 'naive' investors by manufacturing portfolios with same expected loss as generic AAA, but different systematic risk and selling them at identical prices.
- CJS find evidence for their story using CDX.IG synthetic tranche prices:
  - Use pricing model for tranches based on the one-factor Gaussian copula market standard.
  - Instead of assuming that the common factor has a Gaussian density (as in the standard model), the authors extract its density from long-term S&P500 option prices.
  - Their results suggest that observed market spreads on all mezzanine and senior tranches are substantially lower than model-implied 'fair' spreads.

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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### Overview and main results of our paper

- Revisit the relative pricing of tranches and SP500 options
- Same market: CDX-IG tranches
- Propose a structural model to price both SP500 options and CDO tranches written on portfolio of single names.
- Allows us to model the dynamics of default and investigate the term structure of credit spreads.
- Main findings:
  - The model consistently prices tranches and options when calibrated:
    - to SP500 options to match market dynamics (systematic risk).
    - to the term structure of credit spreads to capture idiosyncratic dynamics.
  - Timing of default has first order impact on tranche spreads (especially on difference between equity and senior tranches). This cannot be captured in a one-period model.
  - The ratio of idiosyncratic to market wide jump risk is crucial to capture the tail properties of the loss distribution.
  - Quoted index options are not informative about pricing of senior tranches (too 'narrow' strike range). Difficult to extrapolate much about fair-pricing of AAA tranches based on quoted SP500 options.

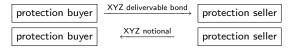
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# The CDX index

- The CDX index is an insurance contract against credit events of a portfolio of counterparties (e.g., 125 names in CDX.IG):
  - Prior to credit event:



Upon arrival of credit event of XYZ:



- Following credit event outstanding notional is reduced by notional of XYZ in portfolio (i.e., 1/125 in CDX.IG).
- Contract expires at maturity or when notional exhausted.
- ▶ N.B.: CDX contract  $\approx$  equally weighted portfolio of single name CDS contracts CDX spread  $\approx$  average of single name CDS spreads

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# Synthetic CDO Tranches

- Selling protection on CDO tranche with attachment points [L, U] (i.e., notional = U - L) written on underlying basket of 125 single names (CDX):
  - Prior to a credit event:



▶ Upon arrival of credit event (LGD = notional - deliverable bond price), if cumulative loss exceeds lower attachment point (i.e.,  $\mathcal{L}_t = \sum_{i=1}^{125} LGD_i \mathbf{1}_{\{\tau_i < t\}} > L$ ) then

- Following credit event outstanding tranche notional is reduced by LGD (up to exhaustion of outstanding notional).
- Also, super senior tranche notional is reduced by recovery (to satisfy 'adding up constraint').
- Contract expires at maturity or when tranche notional is exhausted.
- ▶ Tranche payoff is call spread on cumulative loss:  $\max(\mathcal{L}_t L, 0) \max(\mathcal{L}_t U, 0)$ .
- $\Rightarrow$  Tranche valuation depends on entire distribution of cumulative portfolio losses and crucially on default event correlation model.

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# Market Model: Implied Gaussian Copula Correlation

- Market standard for quoting CDO tranche prices is the *implied correlation* of the Gaussian Copula framework.
- Intuition builds on structural model of default (CDO model due to Vasicek 1987 who combines Merton (1974) with CAPM idea):
  - Each name in basket characterized by an 'asset value' driven by two factors: a common market factor and an idiosyncratic factor

 $(V_i = \beta_i M + \sqrt{1 - \beta_i^2} \epsilon_i \text{ with } M, \epsilon_i \text{ independent centered Gaussian}).$ 

- Pairwise 'asset correlation' is the product of the individual asset betas ( $\rho_{ij} = \beta_i \beta_j$ ).
- Default occurs when asset value falls below a constant barrier (DefProb =  $P(V_i \leq B_i)$ ).
- ▶ Market convention for quoting tranche values in terms of *implied correlation* assumes:
  - The individual beta is identical across all names in the basket.
  - The default boundary is identical and calibrated to CDX level.
  - All firms have identical LGD of 60%.
- $\Rightarrow$  With these heroic assumptions, a single number, the *implied correlation* (=  $\rho$ ), allows to match a given tranche's model price with the market price (for a given CDX level).

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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#### The implied correlation smile

Market Quotes on Aug. 4, 2004 (CDX index spread 63.25 bp)

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG (bps)	4138	349	135	46	14

The market displays an implied correlation smile:

Imp Corr 21.7% 4.1% 17.8%	% 18.5% 29.8%
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 $\Rightarrow$  The smile shows that the Gaussian copula model is mis-specified ( $\sim$  option skew).

Market quotes on June 1st 2005 IG4-5Y (CDX index spread of 42 bp):

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG	3050	66	9.5	7.5	4
Imp Corr	9.08%	5.8%	10.02%	16.77%	27.62%

Market quotes on June 4, 2008 IG9-5Y (CDX index ref 118 bp):

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
CDX.IG	5150	435	232	130	70	41
Imp Corr	40%	88.23%	4.31%	13.47%	32.06%	88.35%

Motivation Overview	CDX Market	The model	Results	Final Thoughts
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A structural model for pricing long-dated S&P500 options

The market model is the Stochastic Volatility Common Jump (SVCJ) model of Broadie, Chernov, Johannes (2009):

$$\begin{aligned} \frac{dM_t}{M_t} &= (r-\delta) dt + \sqrt{V_t} dw_1^Q + (e^y - 1) dq - \bar{\mu}_y \lambda^Q dt - (e^{y_c} - 1) (dq_c - \lambda_c^Q dt) \\ dV_t &= \kappa_v (\bar{V} - V_t) dt + \sigma_v \sqrt{V_t} (\rho dw_1^Q + \sqrt{1 - \rho^2} dw_2^Q) + y_v dq \\ d\delta_t &= \kappa_\delta (\bar{\delta} - \delta_t) dt + \sigma_\delta \sqrt{V_t} (\rho_1 dw_1^Q + \rho_2 dw_2^Q + \sqrt{1 - \rho_1^2 - \rho_2^2} dw_3^Q) + y_\delta dq. \end{aligned}$$

- ▶ We add stochastic dividend yield (SVDCJ) to be help fit long-dated options as well.
- The parameters of the model are calibrated to 5-year index option prices obtained from CJS.
- State variables are extracted given parameters from time-series of short maturity options (obtained from OptionMetrics).
- Advantage of using structural model: Arbitrage-free extrapolation into lower strikes (needed for senior tranches).

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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### Calibration of option pricing model to long-dated S&P500 options

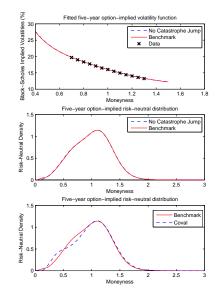
	Pre-crisis (<	Sept. 2007)	Post	-Crisis
Parameter	Estimation 1	Estimation 2	Estimation 3	Estimation 4
ρ	-0.48	-0.48	-0.48	-0.48
$\sigma_V$	0.2016	0.2016	0.2016	0.2016
$\lambda$	0.1534	0.1608	0.1743	0.2465
$\rho_q$	0.0203	0.0199	-0.0509	-0.0576
$\mu_y$	-0.2991	-0.2843	-0.4726	-0.3479
	0.2445	0.2441	0.4609	0.3915
$\vec{v}_y$ $\vec{V}$	0.0037	0.0038	0.0132	0.0094
$\mu_V$	0.0035	0.0033	0.0099	0.0056
$\kappa_V$	5.4368	5.3644	1.5442	2.1596
$V_0$	0.0037	0.0038	0.0132	0.0094
$\kappa_{\delta}$	-0.5914	-0.5903	-0.4816	-0.4953
$\overline{\delta}$	0.04	0.04	0.04	0.04
$\sigma_{\delta}$	0.0454	0.0423	0.0405	0.0304
$\rho_1$	-0.9054	-0.8968	-0.5056	-0.4135
$\rho_2$	-0.0032	-0.0036	-0.0078	-0.0066
$\mu_d$	0.0002	0.0002	0.003	0.0007
$\sigma_d$	0.0007	0.0008	0.0006	0.0006
$\delta_0$	0.04	0.04	0.04	0.04
r	0.05	0.05	0.05	0.05
Уc	0	-2	0	-2
$y_{c}$ $\lambda_{c}^{Q}$	0	0.00076	0	0.0066

Excellent fit

▶ Note: (risk-neutral) mean-reversion coefficient on dividend yield negative.

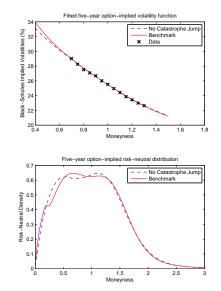
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# Pre-crisis Option pricing fit



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# During-Crisis Option pricing fit



Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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### A structural model of individual firm's default

• Given market dynamics, we assume individual firm *i* dynamics:

$$\begin{aligned} \frac{dA_i(t)}{A_i(t)} + \delta_A dt - rdt &= \beta_i \left( \sqrt{V_t} dw_1^Q + (e^y - 1) dq - \bar{\mu}_y \lambda^Q dt \right) + \sigma_i dw_i \\ &+ (e^{y_c} - 1) \left( dq_c - \lambda_c^Q dt \right) + (e^{y_i} - 1) \left( dq_i - \lambda_i^Q dt \right). \end{aligned}$$

Note

- $\beta$ : exposure to market excess return (i.e., systematic diffusion and jumps).
- *dq<sub>C</sub>*: 'catastrophic' market wide jumps.
- *dq<sub>i</sub>*: idiosyncratic firm specific jumps.
- dw<sub>i</sub>: idiosyncratic diffusion risks.

• Default occurs the first time firm value falls below a default barrier  $B_i$  (Black (1976)):

$$\tau_i = \inf\{t : A_i(t) \le B_i\}. \tag{1}$$

• Recovery upon default is a fraction of the remaining asset value:  $(1 - \ell)B_i$ .

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Pricing of the CDX index via Monte-Carlo

- The running spread on the CDX index is closely related to a weighted average of CDS spreads.
- ▶ Determined such that the present value of the **protection leg**  $(V_{idx,prot})$  equals the PV of the **premium leg**  $(V_{idx,pret})$ :

$$V_{idx,prem}(S) = S E \left[ \sum_{m=1}^{M} e^{-rt_m} (1 - n(t_m)) \Delta + \int_{t_{m-1}}^{t_m} du \, e^{-ru} (u - t_{m-1}) \, dn_u \right]$$
$$V_{idx,prot} = E \left[ \int_0^T e^{-rt} \, dL_t \right].$$

- We have defined:
  - The (percentage) defaulted notional in the portfolio:  $n(t) = \frac{1}{N} \sum_{i} \mathbf{1}_{\{\tau_i < t\}}$
  - ► The cumulative (percentage) loss in the portfolio:  $L(t) = \frac{1}{N} \sum_{i} \mathbf{1}_{\{\tau_i \leq t\}} (1 R_i(\tau_i))$

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Pricing of the CDX Tranches via Monte-Carlo

The tranche loss as a function of portfolio loss is

$$T_j(L(t)) = \max \left[ L(t) - K_{j-1}, 0 
ight] - \max \left[ L(t) - K_j, 0 
ight].$$

The initial value of the protection leg on tranche-j is

$$Prot_{j}(0,T) = \mathsf{E}^{Q}\left[\int_{0}^{T} e^{-rt} dT_{j}(L(t))\right]$$

For a tranche spread  $S_i$ , the initial value of the premium leg on tranche-*j* is

$$Prem_{j}(0, T) = S_{j} E^{Q} \left[ \sum_{m=1}^{M} e^{-rt_{m}} \int_{t_{m-1}}^{t_{m}} du \left( K_{j} - K_{j-1} - T_{j}(L(u)) \right) \right].$$

Appropriate modifications to the cash-flows

- Equity tranche (upfront payment),
- Super-senior tranche (recovery accounting).

		The model	Results	Final Thoughts
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### Calibration of firms' asset value processes

- Calibrate 7 (unlevered) asset value parameters (β, σ, B, λ<sub>1</sub>, λ<sub>2</sub>, λ<sub>3</sub>, λ<sub>4</sub>) to match median CDX-series firm's:
  - Market beta
  - Idiosyncratic risk (estimated from rolling regressions for CDX series constituents using CRSP-Compustat)
  - Term structure of CDX spreads (1 to 5 year)
- ▶ Set jump size to -2 (~ jump to default).
- Calibrate catastrophic jump intensity λ<sub>C</sub> = 0.00076 (less than 1 event per 1000 years) to match super-senior tranche spread (or set to zero for comparison).
- Set loss given default  $\ell$  to 40% ( $\sim$  match historical average) in normal times.
- Set  $\ell = 20\%$  if catastrophe jump occurs ( $\sim$  Altman et al.).
- Market volatility, jump-risk, dividend-yield all estimated from S&P500 option data in previous step.

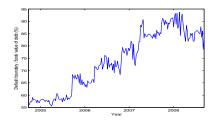
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### Results of Calibration

Systematic risk increased a lot:

Series	Period	Equity Beta	Leverage Ratio	Market Volatility	Idiosyncratic Asset Volatility
3	9/2004-3/2005	0.82	0.36	10.34	27.08
4	3/2005-9/2005	0.83	0.36	10.38	25.29
5	9/2005-3/2006	0.87	0.33	10.02	23.86
6	3/2006-9/2006	0.92	0.33	11.35	21.84
7	9/2006-3/2007	0.94	0.32	9.80	20.93
8	3/2007-9/2007	0.94	0.32	15.67	19.90
9	9/2007-3/2008	0.98	0.31	21.86	18.64
10	3/2008-9/2008	0.99	0.29	23.42	18.61

 Estimates of default boundary rise from 57% to almost 95% (Davydenko (2008), Leland (2004) estimate range (56%, 70%) pre-crisis).



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# Average tranche spreads predicted for pre-crisis period

- ▶ We report six tranche spreads averaged over the pre-crisis period Sep 04 Sep 07:
  - The historical values;
  - Benchmark model: Catastrophic jumps calibrated to match the super-senior tranche; Idiosyncratic jumps and default boundary calibrated to match the 1 to 5 year CDX index.
  - \$\lambda\_c^Q = 0\$: No catastrophic jumps; Idiosyncratic jumps and default boundary calibrated to match 1 to 5 year CDX index;
  - λ<sup>Q</sup> = 0: Catastrophic jumps calibrated to match the super-senior tranche; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index.
  - $\lambda_{C}^{Q} = 0$ ,  $\lambda_{Q}^{Q} = 0$ : No catastrophic jumps; No idiosyncratic jumps; Default boundary calibrated to match only the 5Y CDX index;
  - The results reported by CJS

	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%	0-3% Upfrt
data	1472	135	37	17	8	4	0.34
benchmark	1449	113	25	13	8	4	0.33
$\lambda_c^Q = 0$	1669	133	21	6	1	0	0.40
$\lambda_i^Q = 0$	1077	206	70	32	12	4	0.22
$\lambda_{C}^{Q} = 0, \ \lambda_{C}^{Q} = 0$	1184	238	79	31	6	0	0.26
ັ້	914	267	150	87	28	1	na
CJS — Data Benchmark — Data	24.3	6	9.4	17.5	$\infty$	$\infty$	

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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#### Interpretation

- Errors are an order of magnitude smaller than those reported by CJS.
- However, model without jumps ( $\lambda_c^Q = 0$ ,  $\lambda_i^Q = 0$ ) generates similar predictions to CJS.
- Why? Problem is two-fold:
  - Backloading of defaults in standard diffusion model:

Average CDA index spreads for different models							
	1 year	2 year	3 year	4 year	5 year		
Data	13	20	28	36	45		
Benchmark	13	20	28	36	45		
$\lambda_{C}^{Q} = 0$	13	20	28	36	45		
$\lambda_i^{Q} = 0$	6	7	16	29	45		
$(\lambda_C^{'Q} = 0, \ \lambda_i^Q = 0)$	0	3	13	28	45		

Average CDX index spreads for different models

Idiosyncratic jumps generates a five-year loss distribution that is more peaked around the risk-neutral expected losses of 2.4%.
 (loss distribution with λ<sup>Q</sup><sub>c</sub> = 0, λ<sup>Q</sup><sub>i</sub> = 0 has std dev of 2.9%, whereas loss distribution with (λ<sup>Q</sup><sub>i</sub> > 0, λ<sup>Q</sup><sub>c</sub> = 0) has std dev of 1.7%).

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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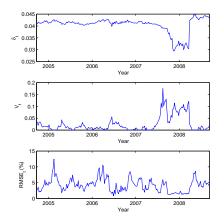
# In Summary:

- In order to estimate tranche spreads, it is necessary that the model be calibrated to match the term structure of credit spreads.
- Specifying a model with idiosyncratic dynamics driven only by diffusive risks generates a model where:
  - the timing of defaults is **backloaded**.
  - $\Rightarrow$  Counter-factually low spreads/losses at short maturities, which biases down the equity tranche spread.
  - the ratio of systematic to idiosyncratic default risk is too high.
  - $\Rightarrow$  Excessively fat-tailed loss distribution, which biases senior tranche spreads up.
- In addition, the super-senior tranche spread (and therefore, spreads on other senior tranches) cannot be *extrapolated* from option prices alone.
- ▶ However, spreads on other tranches can be *interpolated* reasonably well given option prices and super-senior tranche spreads.
- S&P 500 options and CDX tranche prices market can be fairly well reconciled within our arbitrage-free model.

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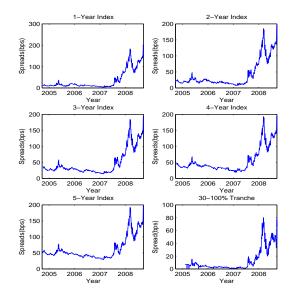
### **Time Series Results**

- Keeping parameters of the option pricing model fixed, each week, we fit
  - the state variables  $V_t$  and  $\delta_t$  to match quoted option prices.
  - > The intensity of the catastrophic jump to match the super-senior tranche,
  - The default barrier and idiosyncratic jump intensity parameters to match the term structure of CDX index spreads with maturities of one-year to five-years.



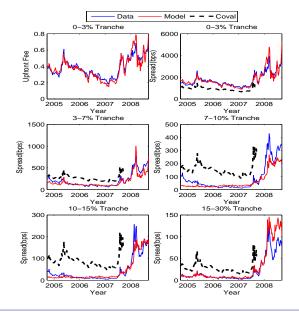
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Series that we match 'in-sample' in benchmark model



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### 'Out of Sample' Time Series Predictions of benchmark model



Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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### **Robustness Analysis**

- ► We study the effects of relaxing some of our simplifying assumptions:
  - firm homogeneity,
  - no changes in capital structure,
  - uncorrelated idiosyncratic shocks (i.e., "no industry effects"),
  - constant firm-level asset dividend yield, (with stochastic market equity dividend yield)
  - constant interest rates.
- ▶ We still calibrate the model to 5-year option implied volatilities, 1-5 year CDX indices, and the super-senior tranche spreads.

	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%	0-3% Upf
data	1472	135	37	17	8	4	0.34
benchmark	1449	113	25	13	8	4	0.33
Dynamic capital structure	1452	116	27	14	8	4	0.34
Stochastic firm payout	1441	122	29	14	9	4	0.33
SVCJ	1330	138	47	26	12	4	0.30
Heterogeneous initial credit spreads	1406	133	28	13	8	4	0.32
Stochastic short-term rate	1484	114	22	11	8	4	0.36
Industry Correlations	1370	153	31	16	10	5	0.31

Table: Robustness check

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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# Details of robustness checks

- Dynamic capital structure: We assume that if a firm performs well, it will issue additional debt, in turn raising the default boundary A<sub>B</sub>(t + dt) = max[A<sub>B</sub>(t), c A(t)]
- ► Stochastic asset dividend yield at the firm level: We specify the firm payout ratio as  $\delta_A(t) = \overline{\delta}_A + \xi(\delta_t \overline{\delta})$ , where  $\overline{\delta}_A = 0.05$  is the average payout ratio, and  $\xi = 0.7$  measures the correlation of dynamics of the firm payout ratio and market dividend-price ratio.
- Constant market equity dividend yield: we specify market dynamics using the SVCJ option model so that both the market dividend price ratio and the firm payout ratio are constants in this scenario.
- Heterogeneity in initial credit spreads: We use our model to back out the default boundaries for each firm based on their average 5-year CDS spreads in the on-the-run period of Series 4. The 5-year CDS spreads are from Datastream. The cross-sectional mean and the standard deviation of the log default boundaries are -1.59 and 0.344. we specify a distribution for the log default boundaries of the 125 firms using a normal distribution with the above parameters.
- Stochastic interest rates: We specify the spot rate to follow Vasciek (1977).
- Industry Correlations: we assume that there are approximately two firms per industry with dynamics that are perfectly correlated. As such, instead of modeling 125 firms, we consider only 60 "industries".

Motivation Overv	ew CDX Market	The model	Results	Final Thoughts
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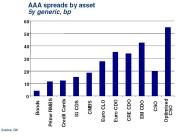
### Conclusion of our analysis

- It is possible to reconcile pricing of SP500 options and CDX-IG tranches within an arbitrage-free structural model of default.
- It is crucial to calibrate the model to the term structure of credit spreads to correctly account for the timing of defaults and the ratio of idiosyncratic to systematic default risk.
- Difference between the equity and senior tranche 'fair spreads' are sensitive to the timing of default. This is not easily captured in a static model.
- ► The ratio of idiosyncratic to systematic default risk varied much during the pre to post crisis period. More systematic risk implied from S&P 500 options actually lead to senior tranche spreads predicted by the model being larger during the crisis than observed (given that the model fits super-senior).
- If anything the model suggests that relative prices of tranches and options were 'more consistent' pre-crisis than during the crisis (in contrast to CJS (2009a,b)).
- Quoted index options are not very informative about pricing of super senior tranches: Quoted strike range is too 'narrow.'
- Caveat: The recalibration of model parameters (default intensities) over time is not internally consistent.

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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### Are senior tranches priced inefficiently by naive investors?

- Investors care only about expected losses (~ ratings) and not about covariance (ironic since they trade in correlation markets!).
- $\Rightarrow$  Spreads across AAA assets should be equalized. Are they?



 $\Rightarrow$  All spreads should converge to **Physical** measure expected loss.

- We observe large risk-premium across the board  $(\lambda^Q/\lambda^P > 6.)$
- Large time-variation in that risk-premium.

 $\Rightarrow$  Time-variation in spreads should be similar to that of rating changes (smoother?).

 Evidence seems inconsistent with marginal price setters caring only about expected loss (~ ratings).

Motivation	Overview	CDX Market	The model	Results	Final Thoughts
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### What drives differences between structured AAA spreads?

- 'Reaching for yield' by rating constrained investors who want to take more risk because their incentives (limited liability) and can because ratings simply do not reflect risk and/or expected loss.
- Taking more risk by loading on systematic risk was the name of the game (agency conflicts).
- Possible that excess 'liquidity'/leverage lead to spreads being 'too' narrow in all markets, but little evidence that markets were ex-ante mis-priced on a relative basis.
- Ex-post (during the crisis) other issues, such as availability of collateral and funding costs, seem more relevant to explain cross-section of spreads across markets.
- Indeed, how to explain negative and persistent:
  - swap spreads?
  - cds basis?