MODEL RISK
or
WARNING: Physics Envy May Be Hazardous To Your Wealth!

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Origins of Modern Economics

- Physics (Samuelson, 1947)
- Samuelson (1998):

Perhaps most relevant of all for the genesis of *Foundations*, Edwin Bidwell Wilson (1879–1964) was at Harvard. Wilson was the great Willard Gibbs's last (and, essentially only) protege at Yale. He was a mathematician, a mathematical physicist, a mathematical statistician, a mathematical economist, a polymath who had done first-class work in many fields of the natural and social sciences. I was perhaps his only disciple… I was vaccinated early to understand that economics and physics could share the same formal mathematical theorems (Euler's theorem on homogeneous functions, Weierstrass's theorems on constrained maxima, Jacobi determinant identities underlying Le Chatelier reactions, etc.), while still not resting on the same empirical foundations and certainties.

Foundations of Modern Economics

Physics Approach In Economics Led To:

- Utility theory, revealed preference (Samuelson)
- General equilibrium theory (Arrow, Debreu)
- Game theory (Harsanyi, Nash, Selten)
- Rational expectations (Lucas, Muth, Sargent)
- Option-pricing theory (Black, Merton, Scholes)
- Efficient markets (Fama, Samuelson)

"Prices fully reflect all available information"

Behavioral Critique of Efficient Markets

- Rationality is not supported by the data
- Cognitive and behavioral biases
  - Loss aversion, anchoring, framing
  - Overconfidence
  - Overreaction
  - Herding
  - Mental accounting

Even Samuelson (1947) Had Reservations:

…[O]nly the smallest fraction of economic writings, theoretical and applied, has been concerned with the derivation of *operationally meaningful* theorems. In part at least this has been the result of the bad methodological preconceptions that economic laws deduced from *a priori* assumptions possessed rigor and validity independently of any empirical human behavior. But only a very few economists have gone so far as this. The majority would have been glad to enunciate meaningful theorems if any had occurred to them. In fact, the literature abounds with false generalization.

We do not have to dig deep to find examples. Literally hundreds of learned papers have been written on the subject of utility. Take a little bad psychology, add a dash of bad philosophy and ethics, and liberal quantities of bad logic, and any economist can prove that the demand curve for a commodity is negatively inclined.

Risk Vs. Uncertainty

Urn A Contains 100 Balls:

- 50 Red, 50 Black
- Pick A Color, Then Draw A Ball
- If You Draw Your Color, $10,000 Prize
- What Color Would You Prefer?
- How Much Would You Pay To Play?

Risk Vs. Uncertainty

Urn B Contains 100 Balls:

- Proportion Unknown
- Pick A Color, Then Draw A Ball
- If You Draw Your Color, $10,000 Prize
- What Color Would You Prefer?
- How Much Would You Pay To Play?

Knight's (1921) Dichotomy of Risk vs. Uncertainty

New Taxonomy of Risk and Uncertainty

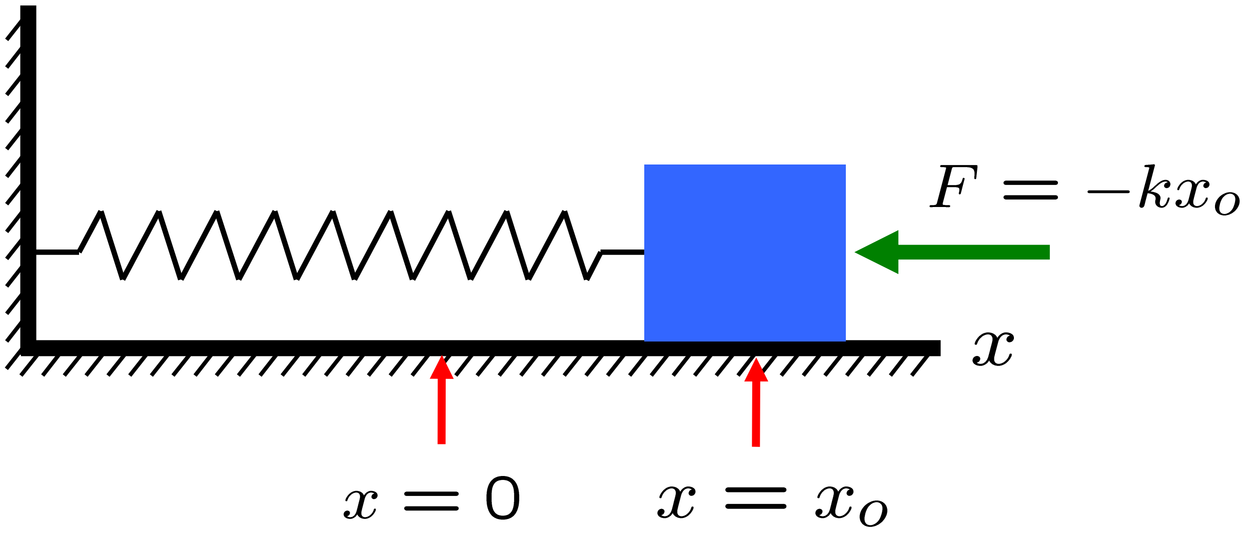
Our Extension of Knight's Dichotomy:

- Level 1: Complete Certainty
- Level 2: Risk without Uncertainty
- Level 3: Fully Reducible Uncertainty
- Level 4: Partially Reducible Uncertainty
- Level 5: Irreducible Uncertainty

An Example: The Harmonic Oscillator

Simplest Non-Trivial Physical Model:

- Motion of an idealized spring without friction
- Hooke's Law: $F = -kx$
- Remarkably powerful and general



An Example: The Harmonic Oscillator

Simplest Non-Trivial Physical Model:

- Apply *F* = *ma* to this relation:

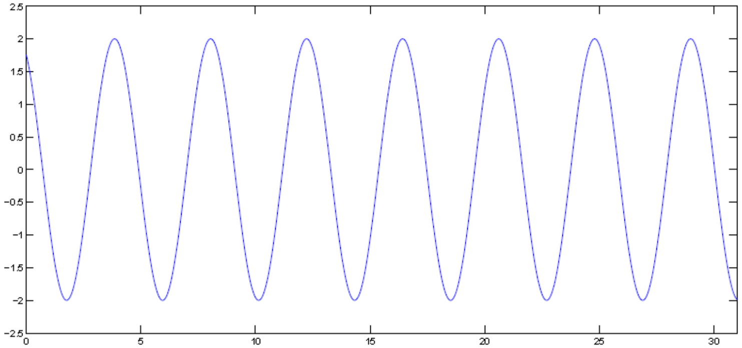
$$0 = \ddot{x} + \frac{k}{m} x$$

$$x(t) = A \cos(\omega_o t + \phi) \quad , \quad \omega_o \equiv \sqrt{k/m}$$

Level 1: Certainty ⇒

- At *t* = 3.5, we **know**

  x = 1.7224



An Example: The Harmonic Oscillator

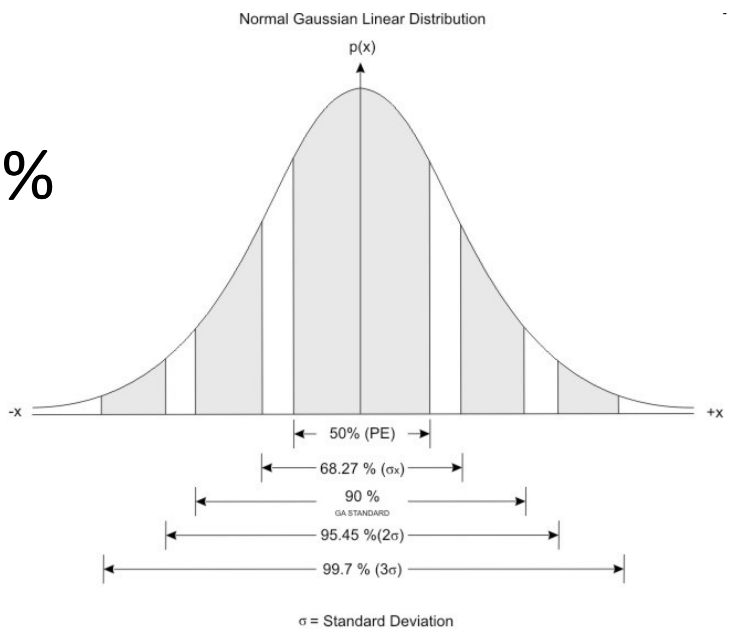
Level 2: Risk without Uncertainty $\Rightarrow$

$$x(t) \;=\; A\cos(\omega_o t + \phi) \;+\; \epsilon(t)$$

$$\epsilon(t) \;\text{IID}\; \mathcal{N}(0, \sigma_\epsilon^2)$$

- At t = 3.5, we **know**

  Prob(x∈[1.4284, 2.0164]) = 5%



An Example: The Harmonic Oscillator

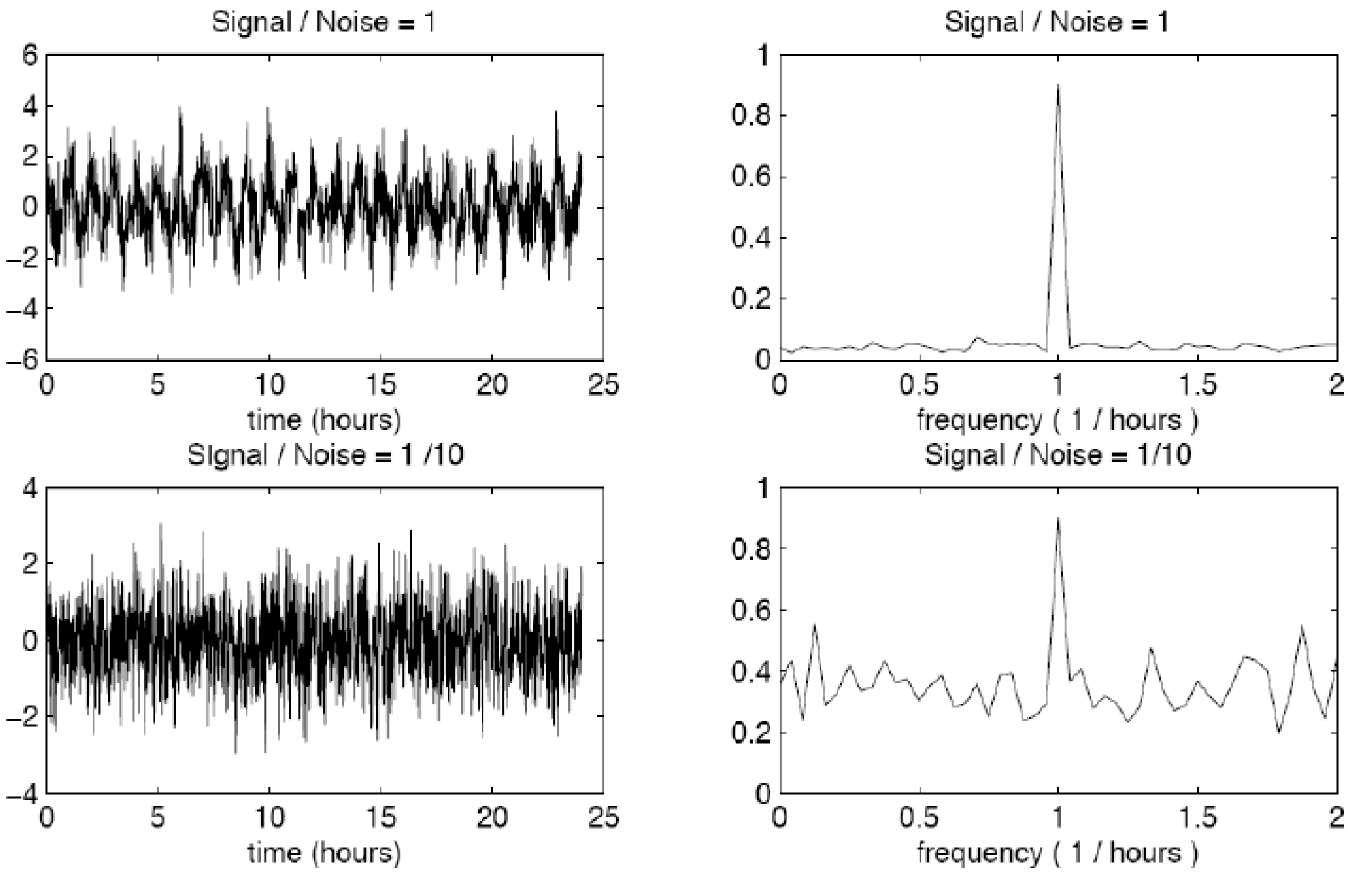
Level 3: Fully Reducible Uncertainty

$$
\begin{aligned}
x(t) &= A\cos(\omega_o t + \phi) \; + \; \epsilon(t) \\
\mathrm{E}[x(t)] &= 0 \\
\mathrm{E}[\epsilon(t)\epsilon(s)] &= \begin{cases} \sigma_\epsilon^2 & \text{if } s \equiv t \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

- Distribution of $\epsilon(t)$ unknown but stationary and ergodic

An Example: The Harmonic Oscillator

Level 3: Fully Reducible Uncertainty $\Rightarrow$



An Example: The Harmonic Oscillator

Level 4: Partially Reducible Uncertainty

- Two-state Markov-switching process
- Observer is unaware of the DGP $\Rightarrow$

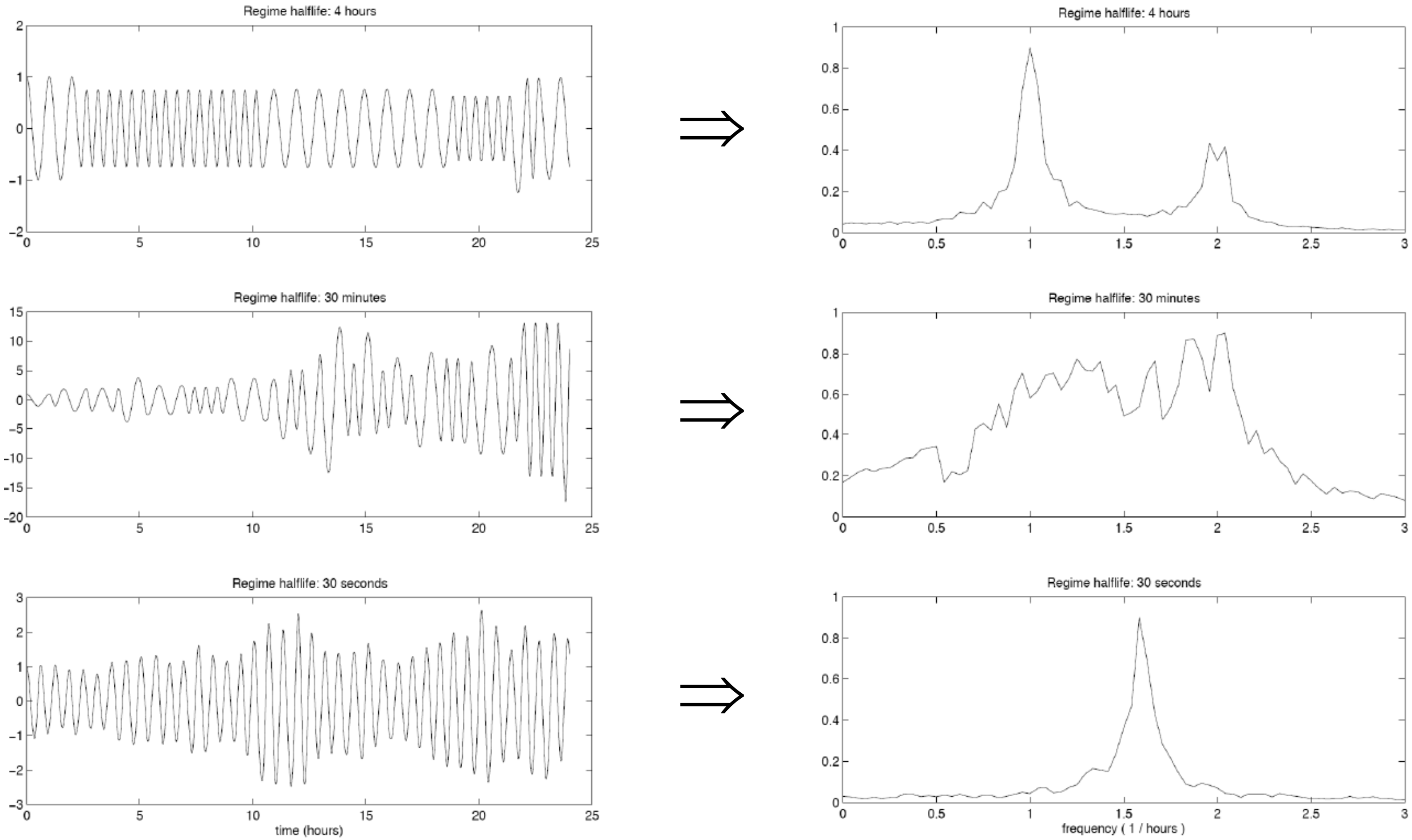
$$x(t) \;=\; I(t)\,x_1(t) \;+\; (1 - I(t))\,x_2(t)$$

$$x_i(t) \;=\; A_i \cos(\omega_i t + \phi_i) \quad , \quad i = 1, 2$$

$$\mathsf{P} \;\equiv\; \begin{array}{c} \\ I(t-1)=1 \\ I(t-1)=0 \end{array} \begin{array}{c} \begin{array}{cc} I(t)=1 & I(t)=0 \end{array} \\ \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix} \end{array}$$

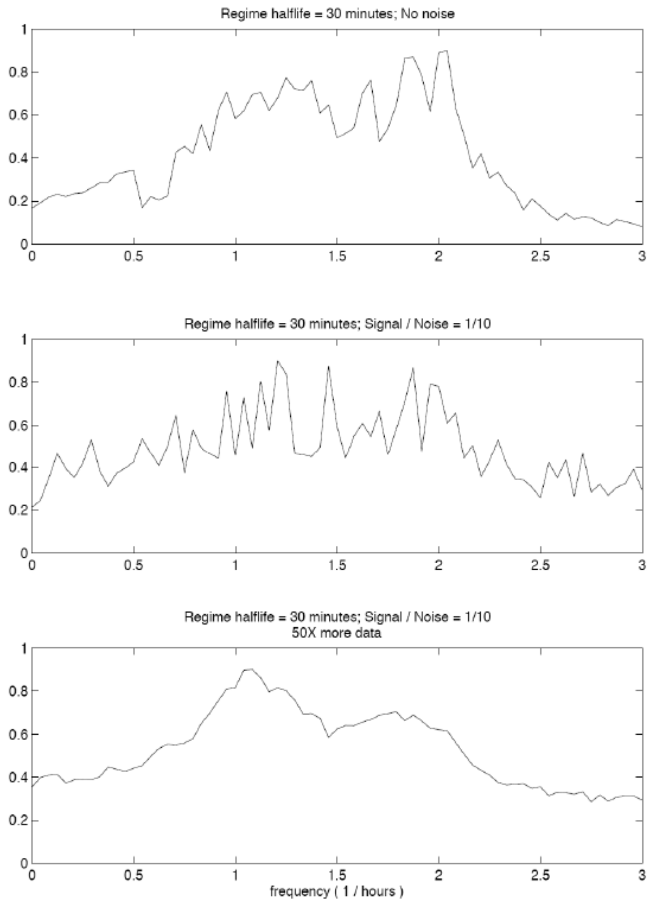
An Example: The Harmonic Oscillator

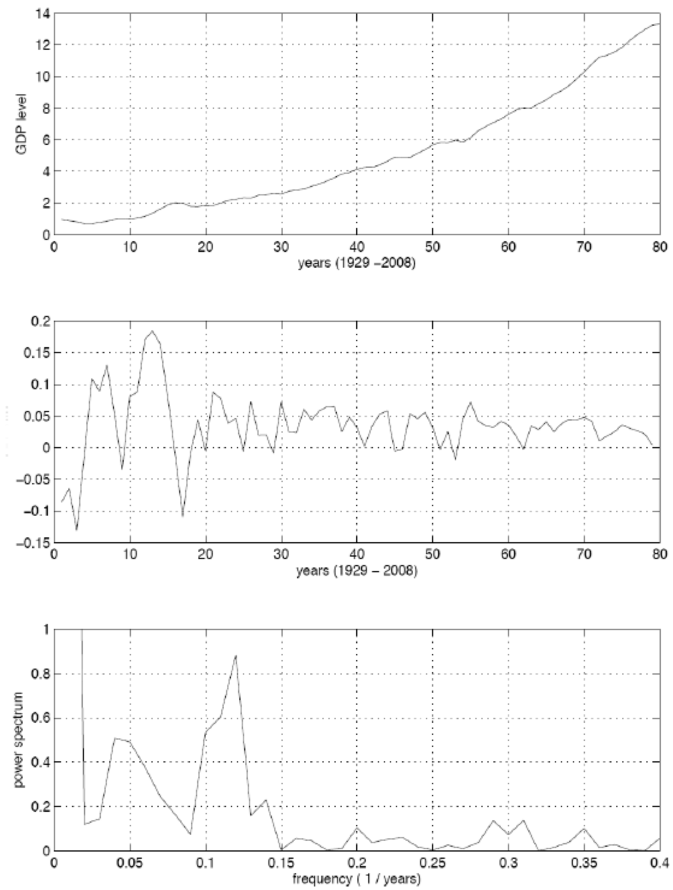
Level 4: Partially Reducible Uncertainty



An Example: The Harmonic Oscillator

Level 4: Partially Reducible Uncertainty





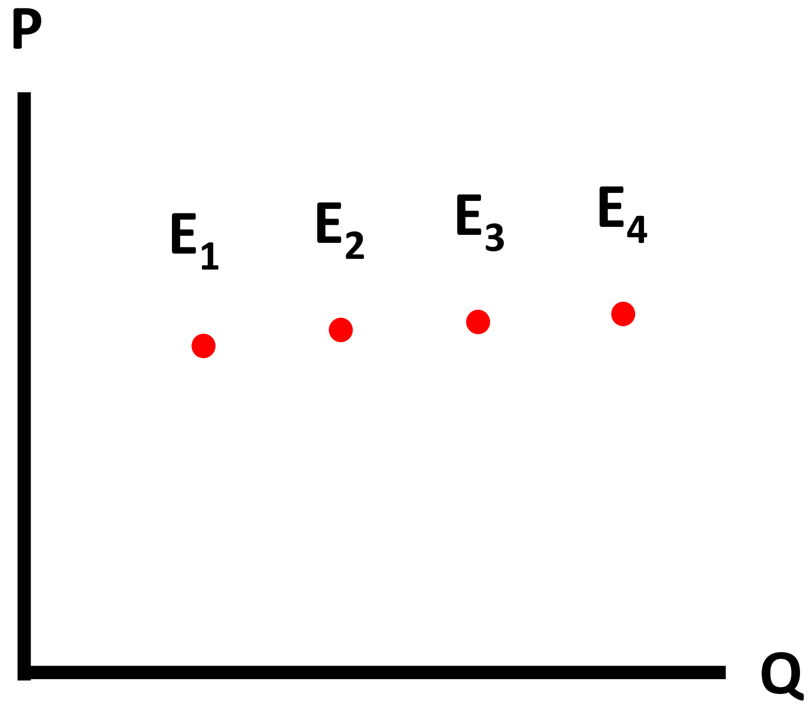
An Example: The Harmonic Oscillator

Level 5: Irreducible Uncertainty (Unknowable)

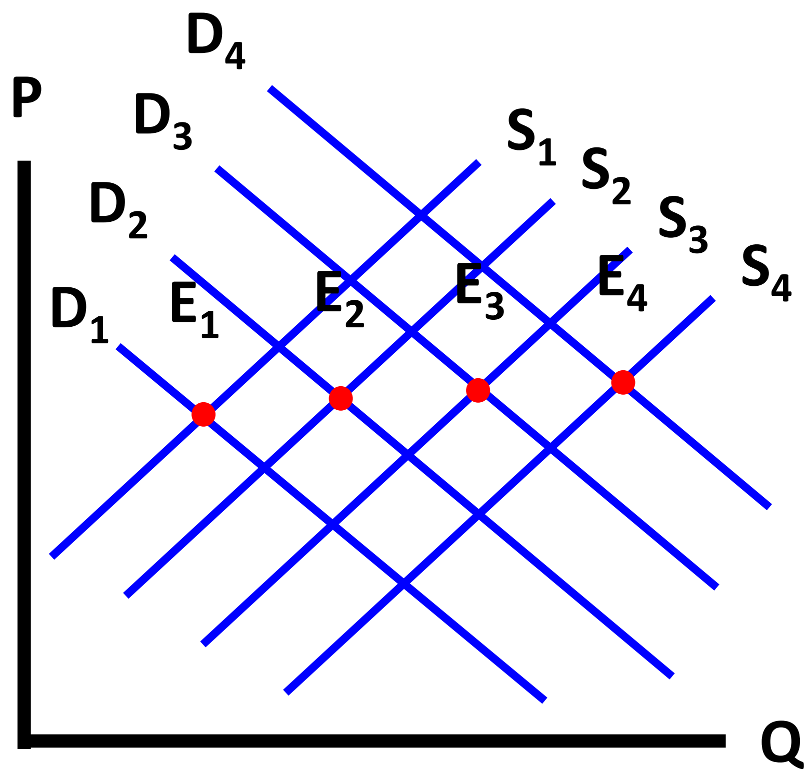
- The “aliasing” or “identification” problem
- Many models may fit the same data, and no possibility of conducting controlled experiments
- This is a major factor in irreducible uncertainty

An Example: The Harmonic Oscillator

Level 5: Irreducible Uncertainty (Unknowable)



Level 5: Irreducible Uncertainty (Unknowable)



The Full Spectrum of Uncertainty

Applies to Fields of Knowledge:

1. Complete Certainty
2. Risk without Uncertainty
3. Fully Reducible Uncertainty
4. Partially Reducible Uncertainty
5. Irreducible Uncertainty

Mathematics
Physics
Chemistry
Biology
Economics
History
Philosophy
Religion

The Full Spectrum of Uncertainty

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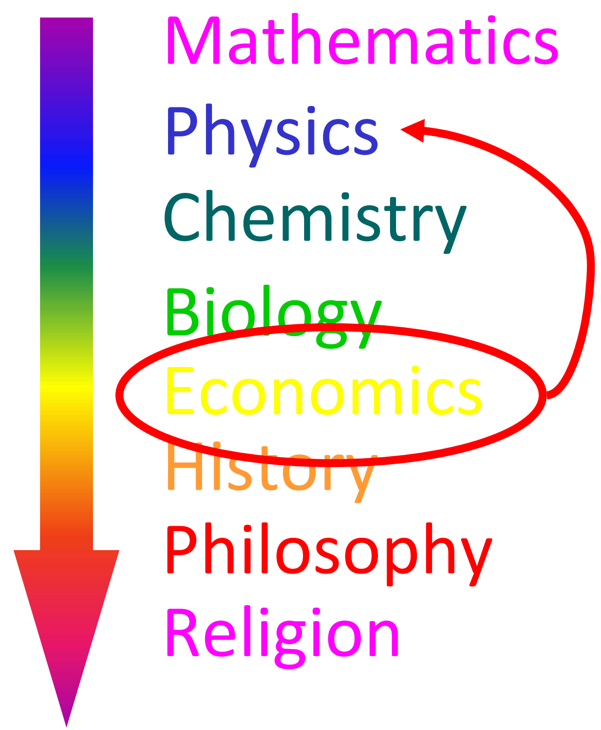

Applying the Taxonomy of Uncertainty

Components of a Quantitative Investment Strategy	Level 1 Perfect Certainty	Level 2 Risk	Level 3 Fully Reducible Uncertainty	Level 4 Partially Reducible Uncertainty	Level 5 Irreducible Uncertainty
Theoretical Framework	Net present value relationships, law of one price	Mathematical framework of mean reversion	Statistical framework of time-series analysis	Unforeseen nonlinearities, omitted variables	Complexity
Empirical Analysis		Econometric estimators and methods of statistical inference	Backtest results based on historical data	Backtest bias, survivorship bias, omitted variables, etc.	Outliers, data errors, insufficient data
Portfolio Construction	Mathematics of optimization	Mean-variance optimization given model parameters	Statistical estimation of model parameters	Time-varying parameters, multiple regimes	Corporate actions, trading halts, shortsales restrictions
Trading and Implementation	Direct trading costs, required technology infrastructure	Probability distributions of trading volume, limit-order fill rates, and market-order impact	Statistical estimation of model parameters	Indirect trading costs (e.g., price impact, opportunity cost), technology and telecom failures	Global flight-to-liquidity, regulatory changes (e.g., shortsales restrictions, ban on flash orders)
Risk Management		Probability theory of loss distributions	Statistical inference for parameters of loss distributions	Time-varying parameters, multiple regimes, and non-stationarities	Tail risk (e.g., terrorism, fraud, flu pandemic)
Business Considerations		Commoditized business services (e.g., market-making, liquidity provision, insurance)	Existing business practices, products, and clients	Near-term business trends, revenue and cost projections, market conditions, re-hypothecation and counterparty risk	Disruptive technologies, global economy-wide shocks, insolvency rumors, flight-to-liquidity
Legal and Regulatory Issues			Existing rules, regulations, and contract terms	Regulatory reform, new tax rules	Government intervention

Conclusion

Physics Envy Can Be Hazardous To Your Wealth

- Consequence of assuming incorrect level of uncertainty
- Multiple levels of uncertainty may apply simultaneously
- Complete risk management protocol includes all levels
- As knowledge accrues, uncertainty decreases
- As expertise departs, uncertainty increases

Do You Know Where Your Uncertainties Are?

Thank You!