Margin-Based Asset Pricing and Deviations from the Law of One Price

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Motivation: Margin-Based Asset Pricing and LoOP

- Key limit of arbitrage: **margin constraints**
- These constraints can become binding; e.g., since 2007:
  - Many traditional liquidity providers have become forced sellers
- One remarkable consequence: Failure of Law of One Price
  - Corporate-bond basis: price gap between bond and CDS
  - Covered interest-rate parity
- Key question: How do margins affect asset prices?

What We Do

- Standard Lucas economy, extended in minimal way:
  - with 2 two agents
  - facing margin constraints
- Derive equilibrium: **Margin CAPM**
- **Quantify** effects of margin
- Help **explain**:
  - CDS-bond basis
  - Failure of covered interest-rate parity (CIP)
  - The effects of the Fed's lending facilities
  - The incentive for regulatory arbitrage

Results: Theory

- Margin (C)CAPM

$$E_t(r^i) - r_t^c \quad = \quad \lambda_t \beta_t^i + \psi_t x_t m_t^i$$

- Shadow cost of capital ψ_t can be captured by
  - interest-rate spreads (LIBOR minus GC repo).
- Binding constraints, $\psi_t > 0$ (e.g., since August 2007):
  - occur following bad fundamental shocks
  - increase Sharpe market ratio: $SR = \bar{SR} + f(x_t)\left(\frac{\bar{SR}}{\bar{\sigma}} - \frac{1}{m}\right)^+$
- Basis: can arise due to difference in margins

$$E_t(r^i) - E_t(r^{i_k}) \quad = \quad \left(\beta_t^{C^b,i} - \beta_t^{C^b,i_k}\right) + \psi_t(m_t^i - m_t^{i_k})$$

- High-margin assets have high sensitivity to funding risk

Results: Applications

- Calibrate model using standard parameters: consumption growth, discount rate, risk aversion, observed margins
  - Large pricing effect of binding constraints
    - Collateralized interest rates drop
    - Interest-rate spreads blow out
    - Margin premium rises
  - High margin assets have high sensitivity to funding risk
    - higher beta
    - higher comovement with each other
- Consistent with model, CDS-bond basis related to:
  - credit tightness (time series)
  - relative margin requirements (cross section)
- Relate interest-rate spread to failure of covered interest parity
- Transmission of unconventional monetary policy:
  - Compute effect of Fed's lending facilities on asset values
- Quantify banks' incentives to loosen capital requirements

Related Literature

- Heterogeneous risk-aversion economies: Dumas (1989); Basak and Cuoco (1998), Chan and Kogan (2002)
- Collateral value: Bernanke and Gertler (1989), Detemple and Murthy (1997), Geanakoplos (1997), Kiyotaki and Moore (1997), Caballero and Krishnamurthy (2001), Lustig and Van Nieuwerburgh (2005), Shleifer and Vishny (2009)
- Equilibrium restrictions with portfolio constraints: Hindy (1995), Hindy and Huang (1995), Cuoco (1997), Aiyagari and Gertler (1999)
- Limits of arbitrage: Shleifer and Vishny (1997), and possible 'arbitrage' in equilibrium: Basak and Croitoru (2000, 2006), Geanakoplos (2003)
- Margin spiral, theory: Brunnermeier and Pedersen (2009); Evidence: Gorton and Metrick (2009)
- Direct evidence from Fed that bids depend significantly on haircuts: Ashcraft, Garleanu, and Pedersen (2009)

Model: Assets

- Continuous-time endowment economy
- Multiple assets in positive supply, characterized by
  - dividend stream: δ_t^i
  - margin requirement: m_t^i
  - endogenous price: $dP_t^i = \left(\mu_t^i P_t^i - \delta_t^i\right) dt + P_t^i \left(\sigma_t^i\right)^\top dB_t$
- Multiple "derivatives":
  - derivative i_k has the same payoffs δ_t^i as asset i
  - smaller margin: $m_t^{i_k} \leq m_t^i$
- Two types of risk-free lending/borrowing:
  - collateralized (rate r_t^c)
  - uncollateralized (rate r_t^u)

Model: Agents

- Two types of agents $g = a, b$:
  - Risk *a*verse: $\gamma^a > 1$
  - Risk tolerant (*b*rave): $\gamma^b = 1$ (i.e., log)
- Utility: constant relative risk aversion

$$\max_{C^g,\theta^i,\eta^u} \mathrm{E}_0 \int_0^\infty e^{-\rho s} \frac{\left(C_s^g\right)^{1-\gamma^g}}{1-\gamma^g}\, ds$$

- Constraints:
  - Solvency: $W_t \geq 0$
  - Funding constraint: $\boxed{\sum_i m_t^i |\theta_t^i| + \eta_t^u \leq 1}$
  - Agent a
    - Does not lend uncollateralized
    - Faces derivative-trading restrictions

Shadow Cost of Capital

Agent b solves

$$\max_{\theta_t^i, \eta_t^u} \left\{ r_t^c + \eta_t^u \left(r_t^u - r_t^c \right) + \sum_i \theta_t^i (\mu_t^i - r_t^c) - \frac{1}{2} \sum_{i,j} \theta_t^i \theta_t^j \sigma_t^i (\sigma_t^j)^\top \right\}$$

subject to $\sum_i m_t^i |\theta_t^i| + \eta_t^u \leq 1$.

Proposition: The shadow cost of the margin constraint is

$$\boxed{\psi_t = r_t^u - r_t^c}$$

Proposition: If agent b is long asset i, its excess return is

$$\boxed{\mu_t^i - r_t^c = \beta_t^{C^b,i} + \psi_t m_t^i} \text{ where } \beta_t^{C^b,i} = \text{cov}_t \left(\frac{dC^b}{C^b}, \frac{dP^i}{P^i} \right)$$

CCAPM with Margins

Suppose that agent a is unconstrained w.r.t. asset i and let

$$\begin{aligned}
\frac{1}{\gamma_t} &= \frac{1}{\gamma^a}\frac{C_t^a}{C_t} + \frac{1}{\gamma^b}\frac{C_t^b}{C_t} \\
x_t &= \frac{\frac{C_t^b}{\gamma^b}}{\frac{C_t^a}{\gamma^a} + \frac{C_t^b}{\gamma^b}} \\
\beta_t^{C,i} &= \mathrm{cov}_t\left(\frac{dC}{C}, \frac{dP^i}{P^i}\right)
\end{aligned}$$

Proposition:

$$\boxed{\mu_t^i - r_t^c = \gamma_t \beta_t^{C,i} + x_t \psi_t m_t^i}$$

CAPM with Margins

Let q be the portfolio with highest correlation with aggregate consumption and

$$\beta_t^i = \frac{\mathrm{cov}_t\left(\frac{dP^i}{P^i}, \frac{dP^q}{P^q}\right)}{\mathrm{var}_t\left(\frac{dP^q}{P^q}\right)}$$

Proposition:

$$\boxed{\mu_t^i - r_t^c = \lambda_t \beta_t^i + x_t \psi_t m_t^i}$$

Basis Trades

Proposition:

- If agent b is long asset i and derivative i_k

$$\boxed{\mu_t^i - \mu_t^{i_k} = \psi_t \left(m_t^i - m_t^{i_k} \right) + \left(\beta_t^{C^b,i} - \beta_t^{C^b,i_k} \right)}$$

- If he is long i and short i_k, then

$$\boxed{\mu_t^i - \mu_t^{i_k} = \psi_t \left(m_t^i + m_t^{i_k} \right) + \left(\beta_t^{C^b,i} - \beta_t^{C^b,i_k} \right)}$$

- The derivative price P^{i_k} decreases with m^{i_k}.

Explicit Equilibrium

Specializing the setup for tractability to consider explicit equilibrium and calibration:

- Aggregate consumption C is geometric Brownian motion
- Continuum of underlying assets with dividend $\delta^i = Cs^i$, where s^i independent martingales
- All underlying assets have the same margin $m^i = m$
- Derivatives with $m^{i_k} \leq m$ traded only by b

Solving Explicitly

- It suffices to calculate equilibrium as if there were one underlying paying C and derivatives on it
- State variables: C and $c^b = C^b/C$
- Pricing kernel for underlying assets: Agent a is marginal:

$$\begin{aligned} \xi_t &= e^{-\rho t}(C^a)^{-\gamma^a} \\ d\xi_t &= \xi_t \left(\mu_t^\xi dt + \sigma_t^\xi dw_t\right) \end{aligned}$$

- Collateralized interest rate:

$$r_t^c = -\mu_t^\xi = -\frac{\mathcal{D}\left(e^{-\rho t}\left(C_t^a\right)^{-\gamma^a}\right)}{e^{-\rho t}\left(C_t^a\right)^{-\gamma^a}}$$

- Market price of aggregate wealth $P_t = C_t \zeta(c_t^b)$:

$$P_t \xi_t = \mathrm{E}_t \int_t^\infty C_s \xi_s \, ds$$

Solution

Proposition:

- Agent b's margin constraint binds iff
$$\frac{\mu - r^c}{\sigma^2} = \frac{SR}{\sigma} \geq \frac{1}{m}$$
- The price-to-dividend ratio $P_t / C_t = \zeta(c_t^b)$ is given as the solution to an ODE and all other endogenous variables are explicit functions of ζ.
- Binding margin constraint increases the Sharpe Ratio:
$$\boxed{SR = \bar{SR} + \frac{x}{1-x} \frac{\bar{\sigma}}{1 - \frac{\zeta' c^b}{m\zeta}} \left(\frac{\bar{SR}}{\bar{\sigma}} - \frac{1}{m} \right)^+}$$
where $\bar{SR} = \gamma \sigma^C$ and $\bar{\sigma}$ are the Sharpe and return volatility without constraints.

Limit Basis

Proposition:

As $c^b \to 0$, the basis between asset i and derivative i_k becomes

$$\mu^i - \mu^{i_k} = \psi(m^i - m^{i_k})$$

where

$$\psi = \frac{\left(\sigma^C\right)^2}{m}\left(\gamma^a - \frac{1}{m}\right)^+$$

In the cross section of asset-derivative pairs,

$$\frac{\mu^i - \mu^{i_k}}{m^i - m^{i_k}} = \frac{\mu^j - \mu^{j_k}}{m^j - m^{j_k}}$$

Calibration: Parameters

- We use the following parameter values

μ^C	σ^C	γ^a	ρ	m	m^{med}	m^{low}
0.03	0.08	8	0.02	0.4	0.3	0.1

- Constraint binds for $c^b \leq 0.22$
- Since b is levered more than a, low c^b is the result of bad shocks to fundamentals

Calibration: Interest Rates

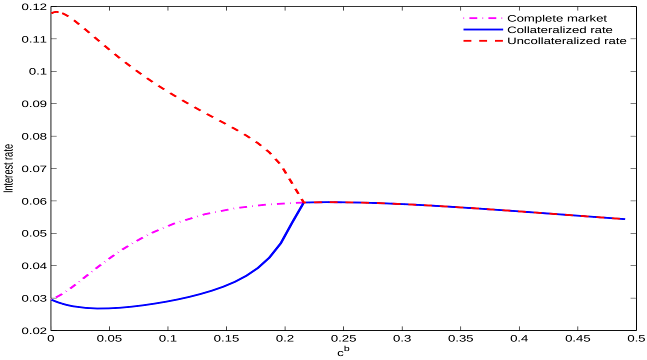


Figure: Interest rates: complete markets, collateralized with constraints (r^c), and uncollateralized with constraints (r^u).

Calibration: Bases

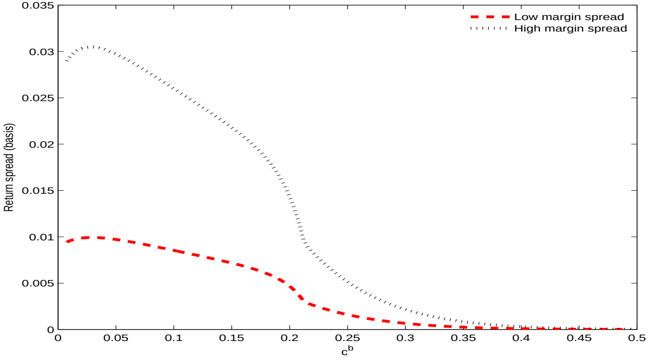


Figure: Return spreads of high-margin underlying versus low-margin derivative (i.e., large margin spread $m^{underlying} - m^{low} = 30\%$) and versus intermediate-margin derivative (i.e., small margin spread $m^{underlying} - m^{medium} = 10\%$).

Calibration: Sharpe Ratios

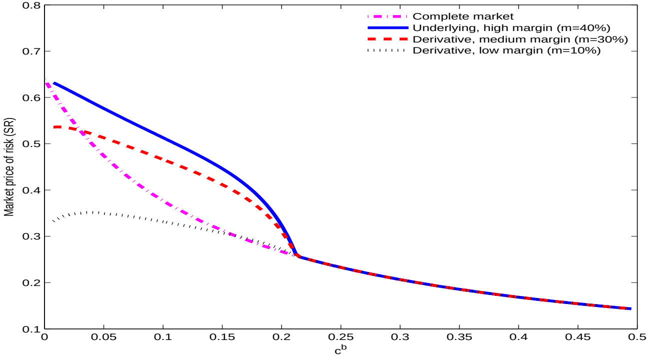


Figure: Sharpe ratios: complete markets, underlying with constraints, and two derivatives with constraints.

Calibration: Price Premium

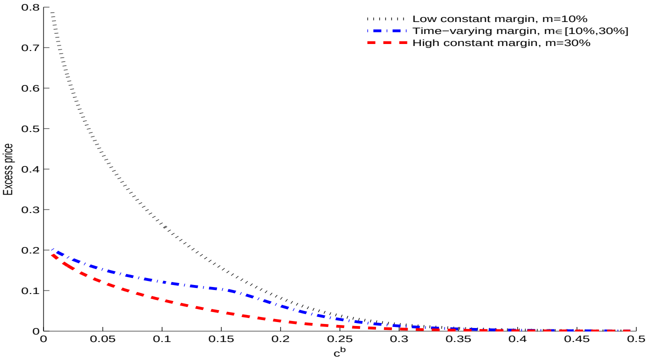


Figure: **Price Premium**. The figure shows how the price premium, $P^{derivative}/P^{underlying} - 1$ for three derivatives with identical cash flows and different margins.

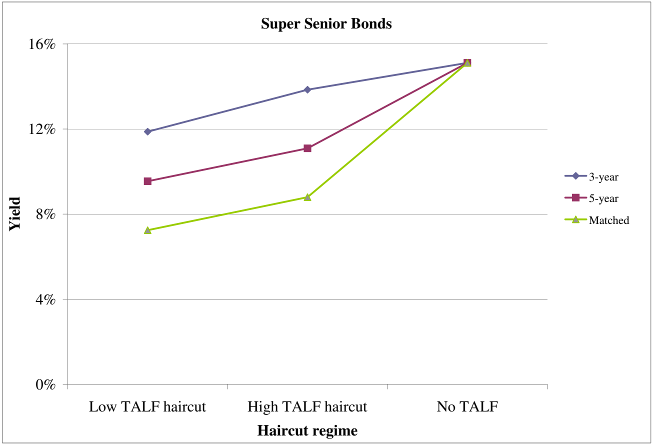
Monetary Policy and Lending Facilities

- Term Auction Facility (TAF) – December 2007
- Term Securities Lending Facility (TSLF) – March 2008
- Term Asset-Backed Securities Loan Facility (TALF) – November 2008
- Goal: Improve funding conditions and "help market participants meet the credit needs of households and small businesses by supporting the issuance of asset-backed securities"
- The model suggests that when the Fed offers lower margins, required returns go down:

$$E(r^{i,Fed}) - E(r^{i,no\ Fed}) \quad \approx \quad \psi(m^{Fed,i} - m^i) \quad < \quad 0$$

- I.e., ABS prices go up, and access to credit eases, helping the real economy

Two Monetary Tools: Interest Rates and Haircuts (Ashcraft, Garleanu, and Pedersen (2009))



Evidence on Monetary Policy and Margins Affecting Prices (Ashcraft, Garleanu, and Pedersen (2009)

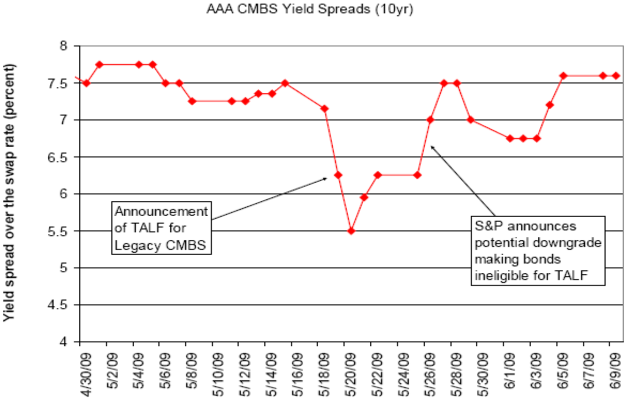


Figure: Market reaction to TALF-related announcements.

CDS-Bond Basis: Time Series

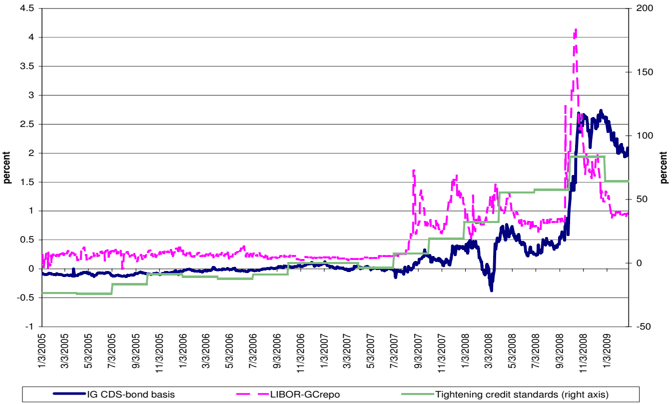


Figure: The CDS-Bond basis, the LIBOR-GCrepo Spread, and Credit Standards.

CDS-Bond Basis: Cross Section

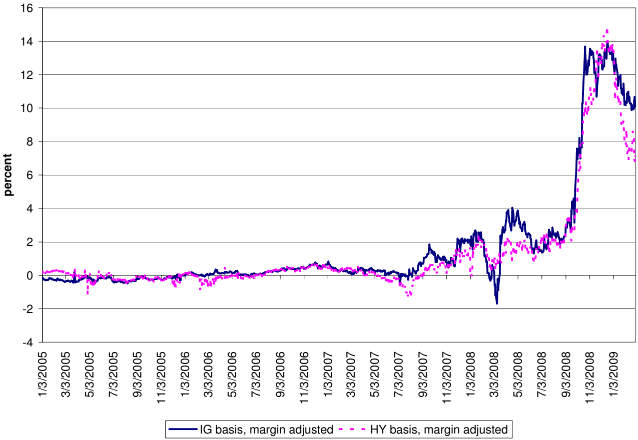


Figure: Investment Grade (IG) and High Yield (HY) CDS-Bond Bases, Adjusted for Their Margins.

Failure of the Covered Interest Rate Parity

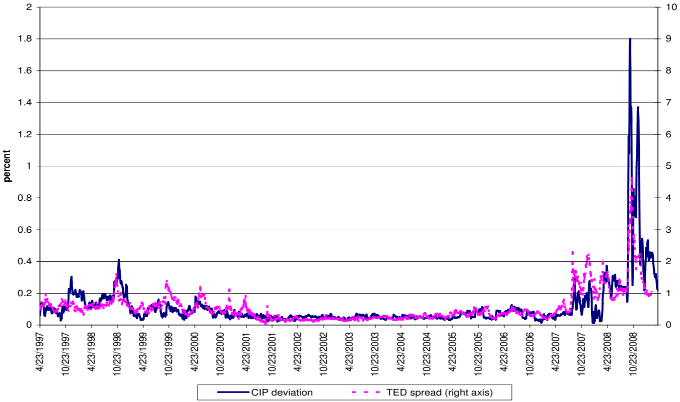


Figure: Average Deviation from Covered-Interest Parity and the TED Spread.

Regulatory Arbitrage

- Pressure to free capital by moving assets off the balance sheet or titling portfolios towards low capital-requirement assets
- Basel requirement is similar to the margin constraint

$$\sum_i m^{Reg,i} |\theta^i| \leq 1$$

- Required return increased by $m^{Reg,i}\psi$

Conclusion

- Margin-based general-equilibrium model
  - Strong asset pricing predictions
  - Predicts that a decline in fundamentals leads to
    - Binding constraints
    - Drop in Treasury and GC repo rates
    - Spikes in interest-rate spreads, risk premium, margin premium
    - Basis between securities with identical cash flows, related to margin differences
- Calibrated model predicts large margin premium in crisis
- Applications:
  - CDS-bond basis
  - Covered interest parity
  - Monetary policy, fed lending facilities
  - Banks' incentives to use off-balance-sheet instruments