A Structural Model of Contingent Bank Capital*

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Abstract

This paper develops a structural credit risk model of a bank that issues deposits, share-holders' equity, and fixed or floating coupon bonds in the form of contingent capital or subordinated debt. The return on the bank's assets follows a jump-diffusion process, and default-free interest rates are stochastic. The equilibrium pricing of the bank's deposits, contingent capital, and shareholders' equity is studied for various parameter values charac-terizing the bank's risk and the contractual terms of its contingent capital. Allowing for the possibility of jumps in the bank's asset value, as might occur during a financial crisis, has distinctive implications for valuing contingent capital. Credit spreads on contingent capital are higher the lower is the value of shareholders' equity at which conversion occurs and the larger is the conversion discount from the bond's par value. The effect of requiring a decline in a financial stock price index for conversion (dual price trigger) is to make contingent capital more similar to non-convertible subordinated debt. The paper also examines the bank's incentive to increase risk when it issues different forms of contingent capital as well as subordinated debt. In general, a bank that issues contingent capital has a moral hazard incentive to raise its assets' risk of jumps, particularly when the value of equity at the con-version threshold is low. However, moral hazard when issuing contingent capital tends to be less than when issuing subordinated debt. Because it reduces effective leverage and the pressure for government bailouts, contingent capital deserves serious consideration as part of a package of reforms that stabilize the financial system and eliminate "Too-Big-to-Fail."

1 Introduction

The recent financial crisis exposed flaws in the regulation of bank capital. At the start of the crisis, there was consensus among U.S. regulatory officials that banks had strong capital positions.[1] Why did these substantial capital levels prove inadequate, leading the federal government to inject more capital into many banks? One explanation is that the crisis produced sudden, extreme losses in bank asset values that rapidly depleted banks' initially high levels of capital. A compounding factor may have been the composition of bank capital. Significant amounts of secondary capital were in the form of subordinated debt.[2] If a bank fails, subordinated debt provides a buffer that protects bank depositors, and therefore the Federal Deposit Insurance Corporation (FDIC), from asset value losses. However since subordinated debt adds to a bank's leverage, it does not reduce the likelihood of financial distress when asset values and primary shareholders' equity capital decline.

Indeed, during the crisis investors gauged a bank's strength as a going concern based on its "tangible common equity" ratio, a capital ratio that excludes subordinated debt. Moreover, because the federal government considered some financial institutions either too big, too complex, or too interconnected to fail (TBTF), it contributed new capital when they became financially distressed, negating subordinated debt's loss-absorbing role at failure. To rectify this shortcoming, a new type of "contingent capital" has been proposed that would decrease TBTF government assistance by reducing the likelihood of financial distress. Contingent capital takes the form of debt that automatically converts to additional common shareholders' equity when a bank's original shareholders' equity is depleted. Banks may prefer to issue contingent capital, rather than an equivalent amount of common shareholders' equity, because contingent capital's status as debt makes it tax-advantaged.[3]

The goal of this paper is to analyze how contingent capital's contractual terms and the risk of

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[1] For example, in September 5, 2007 testimony to the U.S. House of Representatives Financial Services Committee, Federal Deposit Insurance Corporation Chair Sheila Bair stated "Because insured financial institutions entered this period of uncertainty with strong earnings and capital, they are in a better position both to absorb the current stresses and to provide much needed credit as other sources withdraw....As the current period of financial stress began, both the banking industry and the deposit insurance system were sound." In an October 15, 2007 speech to the Economic Club of New York, Federal Reserve Chairman Ben Bernanke stated "Fortunately, the financial system entered the episode of the past few months with strong capital positions and a robust infrastructure. The banking system is healthy."

[2] As of June 2007, suborinated debt equaled 2.2% of total assets and equity capital equaled 8.4 % of total assets for the 20 largest domestic bank holding companies. These are book value figures from Y-9C Reports.

[3] Coupons paid on contingent capital would be deductible from income prior to the calculation of the bank's corporate income taxes whereas dividends paid to shareholders are not. In the absence of major corporate tax reform that would eliminate the disincentive to issue shareholders' equity, contingent capital may be a second-best solution for mitigating this tax distortion and improving financial stability. The alternatives of simply requiring banks to hold more common shareholders' equity capital or of taxing bank debt might worsen other distortions. Specifically, higher corporate taxes paid by banks create incentives to excessively securitize bank loans. Han et al. (2010) provide theory and empirical evidence for this distortion.

its issuing bank affect its value and, therefore, the yields that contingent capital investors would require. The paper also explores the risk-taking incentives of banks that issue different forms of contingent capital and compares them to the risk-taking incentives of banks that issue non-convertible subordinated debt. The forms of contingent capital that are investigated are those proposed by Flannery (2005), Flannery (2009b), and McDonald (2009) for which conversion is tied to the market value of the bank's shareholders' equity. Conversion of contingent capital into new shareholders' equity would occur automatically when the market value of the issuing bank's original shareholders' equity falls below a pre-specified threshold level.[4] Contingent capital would pay coupons and have a fixed maturity such that if the value of the bank's original shareholders' equity did not breach the threshold, contingent capital would mature and could be rolled over into a new issue of contingent capital.

Contingent capital is analyzed in the context of a structural credit risk model of a individual bank. Importantly, the bank's assets may suffer sudden losses in value as might occur during a financial crisis. Specifically, the returns on the bank's assets are assumed to follow a jump-diffusion process. The bank funds these assets by issuing three types of claims: short-term deposits; common shareholders' equity; and bonds in the form of contingent capital or subordinated debt. The model captures other arguably realistic characteristics of banks, such as their ability to increase (*decrease*) their deposit borrowing and leverage as their capital rises (*declines*).[5] It also permits the term structure of default-free interest rates to be stochastic and allows coupons paid by contingent capital (prior to possible conversion) to be either fixed rate or floating rate. Different capital thresholds at which contingent capital converts are considered. Contingent capital also can convert to shareholders' equity at a discount to face (par) value, a contract feature that Flannery (2009a) and McDonald (2009) argue would reduce short-sellers' incentives to manipulate the bank's stock price.

The model leads to a simple formula for the fair credit spread that the bank pays on its deposits. This is possible because deposits are modeled as having a short (instantaneous) maturity, and therefore depositors can suffer losses only if the bank's assets decline suddenly. The simplifying assumption of short-term deposits is not unrealistic, particularly for large banks that are approaching financial distress. During the recent financial crisis, credit risk fears limited many large banks to wholesale deposit funding having very short (overnight) maturities. Hence, the model's derived deposit credit spread can be realistically interpreted as the bank's spread on overnight LIBOR borrowing.

The value of bonds in the form of contingent capital or subordinated debt is calculated using the Monte Carlo valuation approach pioneered by Boyle (1977). This is done by simulating two

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[4] A related proposal that entails more regulatory discretion is Squam Lake Working Group (2009).

[5] Collin-Dufresne and Goldstein (2001) develop a structural credit risk model characterized by a mean-reverting leverage ratio and argue that it better fits corporate credit spreads. Adrian and Shin (2010) document that large financial institutions follow similar behavior.

correlated, risk-neutral stochastic processes: the jump-diffusion process for the bank's assets and the process for the instantaneous-maturity interest rate that determines the default-free term structure. A bond's value is found for a given fixed-coupon rate or floating-coupon spread. By varying this rate or spread, one can determine the fair coupon rate or spread for which the newly-issued bond is valued at par.

The paper's findings confirm some conjectures of prior papers, such as Flannery (2005) and Flannery (2009b). If a bank's asset returns follow a pure diffusion process without jumps, and fixed-coupon contingent capital converts to shareholders' equity at its par value, then contingent capital's new-issue yield-to-maturity (par coupon rate) equals a default-free par rate, such as a Treasury bond yield. But since the possibility of conversion lowers contingent capital's effective maturity, contingent capital's comparable default-free yield is less than that of its stated maturity. Thus, if the term structure of default-free Treasury yields is upward sloping, as it normally is, the yield on contingent capital will be less than that of an equivalent-maturity Treasury bond.[6] However, for the case of contingent capital that pays floating-rate coupons, coupon credit spreads above the short-term, default-free interest rate always will be zero.

When, more realistically, the bank's asset returns incorporate a jump process, contingent capital that is specified to convert at its par value will have a yield that rises above default-free yields. This positive credit spread is due to the potential losses that contingent capital investors would suffer if a sudden decline in the bank's asset value requires conversion at below par value. An implication is that new issue credit spreads on contingent capital rise as the bank's total capital and the value its original shareholders' equity declines. Credit spreads on contingent capital also are higher the lower is the value of shareholders' equity at which conversion is specified to occur and the larger is the conversion discount from the bond's par value.[7] The effect of requiring a decline in a financial stock price index for conversion, the "dual price trigger" feature proposed by McDonald (2009), is to make contingent capital more similar to non-convertible subordinated debt.

The benefits of contingent capital in reducing the likelihood of financial distress may be partially offset if the bank has the incentive to raise the risks of its assets' returns so as to shift risk from the bank's original shareholders to contingent capital investors. If this occurs after contingent capital is issued, value is transferred from contingent capital investors to the original bank shareholders. The paper's results show that, in general, a bank's risk-shifting incentives increase as bank capital declines, and these risk-shifting incentives are greater when contingent capital is designed to convert at a discount from par value or convert at a lower shareholders' equity threshold. However, a bank's risk-shifting incentives when it issues contingent capital

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[6] For example, a contingent capital bond with a maturity of five years may have a coupon lower than that of a five-year Treasury bond.

[7] Specifying conversion at a discount from par value leads to higher yields even in the absence of possible jumps in the bank's asset returns.

tend to be less than those which would occur if the contingent capital were replaced with an equivalent amount of subordinated debt. Thus, if the status quo is one where banks issue subordinated debt, moral hazard may be reduced by substituting contingent capital for subordinated debt.

The following is the plan of the paper. Section 2 introduces a structural model of a banking firm that issues short-term deposits, longer-term (possibly convertible) bonds, and shareholders' equity. It discusses how fair credit spreads on deposits can be derived and how fair new-issue coupon rates or spreads (yields-to-maturity) can be computed for contingent capital or subordinated debt. This section also considers how one might best specify a contingent capital bond's conversion threshold (trigger). Section 3 gives the model's results. It presents comparative statics for the fair fixed-coupon yields or floating-coupon credit spreads of contingent capital and subordinated debt under various assumptions regarding a bank's risk and the contractual terms of the bonds. It also examines the risk-taking incentives of a bank's shareholders when the bank issues different forms of contingent capital or subordinated debt. Section 4 concludes.

2 A Structural Banking Model

This section presents a structural model of an individual bank whose market value of assets follows a jump-diffusion process. The bank funds its assets with short-term deposits, longer-term bonds, and shareholders' equity. The bonds can take the form of contingent capital or non-convertible subordinated debt. Albul et al. (2010) present an alternative structural model for analyzing contingent capital. They extend the model of Leland (1994) to study a financial firm's choice of a straight bond, contingent capital, and shareholders' equity in the presence of corporate taxes and direct costs of bankruptcy.[8] Their focus is on a firm's incentive to issue contingent capital in the context of an optimal capital structure decision, and they also investigate the incentives for stock price manipulation and risk-shifting.

The current paper's model takes a bank's initial capital structure as given and considers how different aspects of the bank's risk and the contractual terms of its contingent capital affect the yields required by investors as well as the risk-shifting incentives of the issuing bank.[9] The distinctive features of the model include: the possibility of jumps in the bank's asset returns; short-maturity deposits; stochastic interest rates; and mean-reverting leverage (capital ratios).

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[8]Both the straight bond and contingent capital are assumed to be consol bonds (perpetuities).

[9]The overlap of the current paper with Albul et al. (2010) is primarily in the examination of risk-shifting incentives. However, since the current paper's model allows for the possibility of jump risk in the bank's asset returns, it draws some additional, distinct conclusions regarding risk-shifting incentives. Section 3.3 elaborates on this issue.

2.1 Assumptions

The model's assumptions relate to the bank's assets and the types of securities that are issued to fund them. Let us start by describing the assets' rate of return process and then discuss the bank's various liabilities.

2.1.1 Bank Assets

A bank's assets are invested in a portfolio of loans, securities, and off-balance sheet positions whose rate of return follows a mixed jump-diffusion process. The date t value of this asset portfolio is denoted A_t. The change in the quantity of bank assets equals the assets' return plus changes due to cash inflows less cash outflows. The sources of inflows and outflows from bank assets will be specified shortly, but for now the superscript $*$ is used to distinguish asset changes solely due to their rate of return, not including changes due to net cash inflows. Thus, the instantaneous rate of return that the bank earns on its assets is denoted as dA_t^*/A_t^*. Under the risk-neutral probability measure, Q, this rate of return follows the process:

$$\frac{dA_t^*}{A_t^*} = (r_t - \lambda_t k_t)\, dt + \sigma\, dz + \left(Y_{q_{t-}} - 1\right) dq_t \tag{1}$$

Note that dz is a standard Brownian motion process under the risk-neutral measure and q_t is a Poisson counting process that increases by 1 whenever a Poisson-distributed event occurs. Specifically, dq_t satisfies

$$dq_t = \begin{cases} 1 & \text{if a jump occurs} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

During each time interval, dt, the risk-neutral probability that q_t augments by 1 is $\lambda_t\, dt$, where λ_t is the risk-neutral Poisson intensity. $\{Y_n\}_{n \in \mathbb{N}}$ is a sequence of random variables such when $q_{t-} = n - 1$ and $q_t = n$, there is a discontinuous change in the bank's assets at date t equal to

$$A_{t+}^* = Y_{q_{t-}} A_{t-}^* \tag{3}$$

where $Y_{q_{t-}}$ is a random variable realized at date t. Thus, if $Y_{q_{t-}}$ is greater (*less*) than one, there is an upward (*downward*) jump in the value of the bank's assets. Define $k_t \equiv E_t^Q[Y_{q_{t-}} - 1]$ as the risk-neutral expected proportional jump in the value of the assets given that a Poisson event occurs. Assuming that the risk-neutral jump probability and jump intensity are independent, the risk-neutral expected change in A^* from the jump component $\left(Y_{q_{t-}} - 1\right) dq_t$ over the time interval dt is $\lambda_t k_t\, dt$.

The sample path of A_t^* for a process described by equation (1) will be continuous most of the time, but can have finite jumps of differing signs and amplitudes at discrete points in time,

where the timing of the jumps depends on the Poisson random variable q_t and the jump sizes depend on the random variable $Y_{q_{t^-}}$. As in Duffie and Lando (2001), these jump events may be interpreted as times when important information affecting the value of the assets is released.

2.1.2 Deposits

In addition to longer-term bonds and shareholders' equity, deposits are one of the bank's three funding sources. The date t quantity of deposits is denoted D_t. Deposits are the most senior claim and are assumed to have an instantaneous maturity; that is, they are short-term or overnight sources of funding for the bank. Deposits pay a competitive return. Some deposits may be fully insured by a government deposit insurer, such as the FDIC, that assesses an instantaneous insurance premium per dollar of deposit, h_t, that fairly reflects its risk-neutral expected insurance claims. In addition, the bank pays interest to the insured depositors at the competitive, instantaneous-maturity default-free rate, r_t. Other deposits may be uninsured and are paid the competitive, default-free interest rate, r_t, plus the fair credit risk premium, h_t. In either the case of insured or uninsured deposits, the bank is assumed to continuously pay out total interest and deposit premiums of $(r_t + h_t) D_t dt$.

With interest and insurance premiums paid out continuously, the bank's total quantity of deposits changes only due to growth in net new deposits (deposit inflows or outflows), which are not directly related to the accrual of interest and premiums. Because empirical evidence, such as Adrian and Shin (2010), finds that banks have target capital ratios and deposit growth expands when banks have excess capital, the model assumes that deposit growth is positively related to the bank's current asset-to-deposit ratio, defined as $x_t \equiv A_t/D_t$. Specifically,

$$\frac{dD_t}{D_t} = g\left(x_t - \widehat{x}\right) dt \tag{4}$$

where g is a positive constant and $\widehat{x} > 1$ is a target asset-to-deposit ratio. When the actual asset-to-deposit ratio exceeds its target, $x_t > \widehat{x}$, the bank issues positive amounts of net new deposits. When $x_t < \widehat{x}$, the bank is gradually shrinking its balance sheet. Thus, the deposit growth rate per unit time, $g\left(x_t - \widehat{x}\right)$, creates a mean-reverting tendency for the bank's asset-to-deposit ratio, x_t.[10]

A bank failure, which leads to the deposit insurer taking control of the bank, is assumed to occur whenever the value of the bank's assets falls to or below the value of total deposits.[11] That is, failure is the date t_f at which $A_{t_f} \leq D_{t_f}$ for the first time, equivalent to $x_{t_f} \leq 1$. When

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[10]It would be straightforward to allow net deposit growth to be stochastic; that is, the model could be generalized to incorporate a separate Brownian motion process in equation (4).

[11]As discussed below, an exception to this closure policy is considered if the bank issues subordinated debt rather than contingent capital. In this case, the bank is assumed to be closed whenever that value of bank assets first falls below the sum of the par values of both deposits and subordinated debt.

failure occurs, the deposit insurer and the uninsured depositors are assumed to proportionally share any loss which totals $D_{t_f} - A_{t_f}$.[12]

The assumptions that deposits have an instantaneous maturity and that a bank is closed whenever the value of assets falls below the promised value of deposits imply that only Poisson jumps can cause bank failure losses to depositors. These assumptions simplify the calculation of the fair credit risk spread on deposits, h_t, and they also simplify the valuation of the bank's other liabilities, including shareholders' equity and contingent capital or subordinated debt. Because deposits are fairly compensated for possible losses at any point in time, changes in bank capital or the design of other liabilities will not shift value from or to depositors. This allows us to examine how changes in capital, bond contract terms, and asset risk affect the relative values of the bank's other liabilities without having to consider value transfers to or from deposits. Moreover, these assumptions regarding deposits may not be a gross departure from reality. For many large banks, especially large banks nearing financial distress, wholesale deposits are indeed typically of short maturity, often overnight Eurodollar deposits paying a rate close to overnight LIBOR.

2.1.3   Contingent Capital

The bank's longer-term bonds may take the form of contingent capital with a design similar to that of Flannery (2009a). Let us consider this case first. Later, bonds in the form of subordinated debt or contingent capital with the dual price trigger design of McDonald (2009) are discussed.

Suppose that a contingent capital bond is issued at date 0 and matures at date $T > 0$. It has a par (principal or face) value of B and continuously pays a coupon equal to $c_t B dt$ as long as the bond is not converted or the bank has not failed. If the contingent capital bond is specified to pay a fixed-rate coupon, then $c_t = c$, a constant. If, instead, it has a floating-rate coupon, then $c_t = r_t + s$, where s is a fixed spread over the short-term (instantaneous-maturity) interest rate. The value of c or s is set at the time of issue (date 0) such that the bond's equilibrium value equals par, B. The bond does not convert (and the bank would not fail) as long as the bank's asset to deposit ratio, x_t, stays above a pre-specified threshold conversion level, $\bar{x}_t > 1$, during the period from 0 to T. Alternatively, the first time x_t takes the value $x_t \leq \bar{x}_t$, say at date t_c, the contingent capital bond converts to common shareholders' equity.[13] This conversion occurs

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[12]Rules guiding the resolution of U.S. bank failures require proportional sharing of losses by uninsured depositors and the FDIC.

[13]Some contingent capital proposals allow for only part of the contingent capital to convert when a threshold is breached. For simplicity, this model assumes that the entire amount of the bank's contingent capital converts to equity. Partial conversion introduces additional complications because the value of shareholders' equity at conversion will depend on the value of unconverted contingent capital, making it more difficult to specify conversion values. However, the model is consistent with the assumption that the bank could be required to

at a number of shares that leaves the value of contingent capital, V_{t_c}, equal to

$$
\begin{aligned}
V_{t_c} &= \begin{cases} pB & \text{if } pB \le A_{t_c} - D_{t_c} \\ A_{t_c} - D_{t_c} & \text{if } 0 < A_{t_c} - D_{t_c} \le pB \\ 0 & \text{if } A_{t_c} - D_{t_c} \le 0 \end{cases} \\
&= [A_{t_c} - D_{t_c}]^+ - [A_{t_c} - D_{t_c} - pB]^+
\end{aligned}
\tag{5}
$$

where the parameter p dictates the maximum conversion value of new shareholders' equity per par value of contingent capital. If $p < 1$ ($p > 1$), bonds are converted to equity at a discount (*premium*) to their par value when the bank has sufficient capital; that is, when $A_{t_c} - D_{t_c} \ge pB$, where $A_t - D_t$ is defined as the bank's total capital at date t.[14]   If capital is positive but insufficient for full conversion, $pB \ge A_{t_c} - D_{t_c} > 0$, then contingent capital converts to an amount of shareholders' equity equal to remaining capital of $A_{t_c} - D_{t_c}$. If $A_{t_c} - D_{t_c} \le 0$, the bank has failed and contingent capital holders receive nothing. Note that bank failure or conversion at less than full value can result only following a downward jump in the bank's asset value that prevents full conversion.

2.1.4 Shareholders' Equity

In addition to deposits and bonds, the bank receives funding from its initial shareholders' equity, whose date t value is denoted E_t. If the bank's bonds take the form of contingent capital and the bank's asset-to-deposit ratio never falls below $\bar{x}_t$ during the period from 0 to T, then the contingent capital never converts, the bank does not fail, and the value of original shareholders' equity is worth $E_T = A_T - B - D_T$ when the contingent capital is paid off in full at date T.[15] Alternatively, the first time x_t takes the value $x_t \le \bar{x}_t$, which was denoted date t_c, then the value of the original shareholders' equity equals

$$
\begin{aligned}
E_{t_c} &= \begin{cases} A_{t_c} - D_{t_c} - pB & \text{if } pB < A_{t_c} - D_{t_c} \\ 0 & \text{if } A_{t_c} - D_{t_c} \le pB \end{cases} \\
&= [A_{t_c} - D_{t_c} - pB]^+
\end{aligned}
\tag{6}
$$

issue new fairly priced contingent capital after the old is converted. This requirement would have no effect on the model's valuation of existing contingent capital.

[14] The paper's analysis focuses on conversion ratios $p \le 1$ since Flannery (2009a) and McDonald (2009) have advocated setting $p < 1$ to prevent manipulation of the bank's stock price in order to force conversion.

[15] As presented, the model assumes that no dividends are paid to shareholders. However, it is straightforward to allow payment of a continuous dividend out of the bank's assets, similar to the way that coupons on bonds and interest on deposits are paid. For example, dividends might be a function of the bank's asset-to-deposit ratio, x_t. The model's qualitative results regarding the pricing of contingent capital and risk-taking incentives would be little changed. Dividend payments would increase the rate at which the bank's assets (and capital) are depleted, thereby leading to somewhat higher fair coupon rates (yields) paid on bonds.

Note that if contingent capital converts or matures, the total value of shareholders' equity (including possibly converted contingent capital) equals the bank's net worth or capital, $A_t - D_t$. At any time afterwards, new contingent capital can be issued at its fair value, B, without any immediate change in the value of existing shareholders' equity. Therefore, the model's valuation of existing contingent capital and shareholders' equity is consistent with any fairly-priced new issue of contingent capital that occurs after the existing issue converts or matures. Thus, the model's valuation of contingent capital and shareholders' equity is consistent with a regulatory requirement that new contingent capital must be issued following the current issue's conversion or maturity. Any subsequent "resetting" of the bank's capital structure, as long as any new security issues are fairly priced, would not affect model's valuation of the bank's current liabilities.

2.1.5 Conversion Threshold

The model can accommodate different specifications of a conversion threshold or boundary, $\overline{x}_t$. Suppose a conversion occurs at date t_h that is exactly at the threshold, so that $x_{t_h} = \overline{x}_{t_h}$. Such a conversion would follow a Brownian motion decline in asset value rather than a Poisson jump that takes x_t strictly below $\overline{x}_t$. One reasonable specification is for the threshold to be in terms of a fixed ratio of assets to deposits $(A_{t_h}/D_{t_h} = \overline{x})$, total capital to deposits $((A_{t_h} - D_{t_h})/D_{t_h} = \overline{x} - 1)$, or total capital to assets $((A_{t_h} - D_{t_h})/A_{t_h} = 1 - 1/\overline{x})$, all of which are effectively equivalent. A slightly different specification would be to state these ratios in terms of capital excluding the par value of contingent capital, such as $(A_{t_h} - B - D_{t_h})/D_{t_h}$ or $(A_{t_h} - B - D_{t_h})/A_{t_h}$.[16]

However, in practice, the bank's asset value or market value of capital is not directly observable. What is observable is the market value of original shareholders' equity, E_t, as well as the par values of contingent capital, B, and deposits, D_t. Hence, it is possible to restate an asset or capital ratio threshold in terms of shareholders' equity if we know the relationship of shareholders' equity to assets at the conversion threshold. This relationship is given by the first line in equation (6). For example, a threshold stated in terms of the ratio of the market value of original shareholders' equity to deposits, say $\overline{e}$, could be stated as

$$\overline{e} = \frac{E_{t_h}}{D_{t_h}} = \frac{A_{t_h} - D_{t_h} - pB}{D_{t_h}} = \overline{x}_{t_h} - 1 - pb_{t_h} \tag{7}$$

where $b_t \equiv B/D_t$ is the ratio of the contingent capital's par value to the date t value of deposits. The equity threshold (7) is equivalent to the asset-to-deposit threshold of $\overline{x}_{t_h} = 1 + \overline{e} + pb_{t_h}$.

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[16]As of this writing, there have been two banks that have issued a type of contingent capital in exchange for non-convertible bank bonds, Lloyds Banking Group and Yorkshire Building Society. In each case the conversion trigger was in terms of a capital-to-asset ratio that excluded contingent capital, namely, a 5% Tier 1 capital asset ratio. However, this conversion threshold was in book value, rather than market value, terms.

For example, if $p = 1$, $b_{t_h} = 4\%$, $\bar{e} = 2\%$, the conversion threshold would be when original shareholders' equity had fallen to 2% of deposits, at which time contingent capital would convert to new equity worth 4% of deposits, so that total capital would be worth 6% of deposits, or assets worth 1.06% of deposits. If $\bar{e} = 2\%$, $b_{t_h} = 4\%$, but $p = 0.9$, so that contingent capital converts at a discount to par value, equal to new equity worth 3.6% of deposits, then total capital would be worth 5.6%, or total assets would be worth 1.056% of assets.

All else equal, if a threshold is stated in terms of the market value of original shareholders' equity and contingent capital converts at a discount to face value, the resulting total capital will be less than if the conversion was at par value. To correct for this, it may make sense to raise $\bar{e}$ to be higher for the case when contingent capital converts at a discount relative to the no-discount case. Thus, in the above example if $\bar{e}_{p=1} = 2\%$, then $\bar{e}_{p=0.9} = 2.4\%$. Making this adjustment, the conversion threshold would always be at the point where assets are 1.06% of deposits.

The next section's comparative analysis makes such an adjustment to keep the total capital to deposit threshold approximately equal for different contingent capital conversion discounts. Thus, the chosen original shareholders' equity to deposit threshold that is set when the contingent capital is issued equals

$$\bar{e}_p = \bar{e}_{p=1} + (1-p)\frac{B}{D_0} = \bar{e}_{p=1} + (1-p)b_0 \tag{8}$$

which is equivalent to the asset to deposit threshold of[17]

$$\bar{x}_{t_h} = 1 + \bar{e}_{p=1} + b_0 + p(b_{t_h} - b_0) \tag{9}$$

Another rationale for making this adjustment is that when contingent capital converts at a discount to par, conversion should occur at a level of total capital exceeding the full face value of contingent capital: $A_{t_h} - D_{t_h} > B$, or $\bar{x}_{t_h} > 1 + b_{t_h}$. Doing so prevents situations where contingent capital has not converted but there is insufficient total capital to pay off its full face value of B at maturity.[18]

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[17] If as in (8) the original shareholders' equity-to-deposit threshold, $\bar{e}_p$, is constant, then (9) shows that $\bar{x}_{t_h}$ is time varying. It would be a constant, equal to $1 + \bar{e}_{p=1} + b_0$ if bank deposits did not vary over time; that is $b_{t_h} = B/D_{t_h} = b_0 = B/D_0$. Since it realistic to permit mean-reversion in capital ratios by allowing deposit issuance to vary, allowing for a time-varying asset-to-deposit ratio conversion threshold would appear to be important given that issuance of new contingent capital (which would change B) would not occur as frequently as new issuance of deposits.

[18] Based on (9), such situations would not occur as long as $\bar{e}_{p=1} > (b_t - b_0)(1-p)$. Because b_t is random, such situations cannot be completely ruled out when $p < 1$. The problem would arise when deposits decline so significantly that the ratio of equity to deposits remains above the threshold but the bank's total capital shrinks below B. For example, if $\bar{e}_{p=1} = 2\%$, $b_0 = 4\%$, and $p = 0.9$, then the value of b_t for which this inequality would not hold would be $b_t = 24\%$. That would represent a 83% decline in deposits, which is probably outside the realm of possibility for a bank that has not yet failed.

If conversion does occur at the threshold value of original shareholders equity of $E_{t_h} = \bar{e}D_{t_h}$, and if there are N original shares of stock, so that $\bar{e}D_{t_h}/N$ is the price per share, then contingent capital worth pB would convert to $pB/\left(\bar{e}D_{t_h}/N\right) = pBN/\left(\bar{e}D_{t_h}\right)$ new shares.

If there is a downward jump in asset and equity values such that $x_t < \bar{x}_t$ or $E_t < \bar{e}D_t$, it may or may not be possible to issue sufficient new shares to make the market value of contingent capital equal pB. If upon issuance of shares equal to pBN/E_t, the value of contingent capital equals its full conversion value of pB, then the original shareholders retain a positive stake in the bank. However, if upon issuance of pBN/E_t shares, the price per share falls to close to zero, this would indicate that contingent capital cannot be converted in full and the original shareholders' stake must be wiped out. If, after giving the previous contingent capital holders full ownership of the bank, the new market value of total equity is very small, this should signal to regulators that there may have been a large enough loss in asset value that capital may even be negative. Such an event should trigger an examination of the bank to determine whether it should be closed.[19]

2.1.6 Subordinated Debt

Rather than contingent capital, suppose that a bank's longer-term bonds take the form of non-convertible, subordinated debt. Using the model to value subordinated debt is useful for comparing the risk-taking incentives of a bank that issues contingent capital to one issuing standard subordinated debt. For this reason, consider how the model's assumptions change when subordinated debt replaces contingent capital. Similar to contingent capital, subordinated debt can be issued at date 0 having a par value of B, a time until maturity of T, and fixed or floating coupons paid out continuously. Suppose that regulators close the bank when assets equal or fall below the total par value of debt, equal to the par value of subordinated debt and deposits; that is, $A_t \leq B + D_t$.[20] Then the value of subordinated debt at a bank's closure will equal

$$V_{t_c}^{sub} = \begin{cases} A_{t_c} - D_{t_c} & \text{if } 0 < A_{t_c} - D_{t_c} \leq B \\ 0 & \text{if } A_{t_c} - D_{t_c} \leq 0 \end{cases} \tag{10}$$

If assets remain above the par values of deposits and subordinated debt, the bank continues to make coupon payments until the bond matures at date T.

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[19] Another indication of whether the bank is still viable would be if it can now issue new contingent capital, which is possible only if current capital is non-negative.

[20] One could assume that regulators do not intervene to close the bank until at least some portion of the subordinated debt is wiped out. For example, the closure rule might be $A_t \leq \delta B + D_t$, where $0 \leq \delta \leq 1$. Such a closure rule would make the subordinated debt even more risky since losses could occur even without a downward jump in asset value.

2.1.7 Contingent Capital with a Dual Price Trigger

The bank's longer-term bonds could take a third general form. As proposed by McDonald (2009), contingent capital with a dual price trigger modifies the design of Flannery (2009b) to impose an additional condition for conversion. Not only must the bank's shareholders' equity fall below a threshold, but an index of financial firms' stocks must also breach a pre-specified threshold. The motivation for including this second condition is to permit conversion, so that a bank remains a going concern, only during a general financial crisis. Instead, if the contingent capital-issuing bank is performing badly while other financial institutions are not, conversion would not occur and the bank could fail. Thus, dual price trigger contingent capital acts like the contingent capital of Flannery (2009b) in a crisis situation but acts like standard subordinated debt in a non-crisis situation.

Let I_t be the date t value of a financial stock index, and let $\overline{I}_t$ be the pre-specified threshold required for conversion. Thus, only if $I_t \leq \overline{I}_t$ and $x_t \leq \overline{x}_t$ would full or partial conversion to shareholders equity occur as in equation (5). If $I_t > \overline{I}_t$ even though $1 + pb_t < x_t \leq \overline{x}_t$, there is no conversion and the bond continues to pay coupons. If $x_t \leq 1 + pb_t$ then regulators are assumed to close the bank and the bond's liquidation value satisfies equation (10).[21]

The risk-neutral process for the financial stock index is assumed to satisfy

$$dI_t/I_t = r_t dt + \sigma_i dz_i \tag{11}$$

where σ_i is a constant and dz_i is a Brownian motion that is correlated with the individual bank's asset return Brownian motion, dz.[22]

Having completed a description of the bank's assets and liabilities, the model's remaining assumption involves the interest rate environment.

2.1.8 Default-Free Term Structure

The model can accommodate different specifications of the default-free term structure. Modeling stochastic interest rates is important for distinguishing between fixed versus floating coupons paid by contingent capital or subordinated debt. For concreteness, consider the term structure

---

[21]Note that for the case of $p < 1$, so that conversion would occur at a discount to par, it is assumed that regulators would not close the bank until $A_t \leq pB + D_t$, rather than $A_t \leq B + D_t$. The logic is that when $pB + D_t < A_t \leq B + D_t$, there is still the possibility that full conversion at pB may occur in the future if I_t later falls below $\overline{I}_t$. However, the model assumes that at maturity an unconverted contingent capital bond will lead to a failure whenever $A_t \leq B + D_t$ since there is insufficient asset value to pay the bond's par value of B.

[22]At the expense of additional parameters, it is straightforward to generalize the index return process (11) to include a Poisson jump component that could be correlated with the individual bank's Poisson jump process. The quantitative effect may be to make conversion more likely, but qualitatively the results will be similar.

specification of Cox et al. (1985) where the risk-neutral process followed by the instantaneous default-free interest rate, r_t, is[23]

$$dr_t = \kappa \left(\bar{r} - r_t \right) dt + \sigma_r \sqrt{r_t} d\zeta \tag{12}$$

and where $d\zeta$ is a Brownian motion process such that $d\zeta dz = \rho dt$. This process implies that the date t price of a default-free, zero-coupon bond that pays \$1 in $\tau \equiv T - t$ years is

$$P\left(r_t, \tau \right) = A\left(\tau \right) e^{-B(\tau) r_t} \tag{13}$$

where

$$A\left(\tau \right) \equiv \left[\frac{2\theta e^{(\theta + \kappa) \frac{\tau}{2}}}{(\theta + \kappa)\left(e^{\theta \tau} - 1 \right) + 2\theta} \right]^{2\kappa \bar{r}/\sigma_r^2}, \tag{14}$$

$$B\left(\tau \right) \equiv \frac{2\left(e^{\theta \tau} - 1 \right)}{(\theta + \kappa)\left(e^{\theta \tau} - 1 \right) + 2\theta}, \tag{15}$$

and $\theta \equiv \sqrt{\kappa^2 + 2\sigma_r^2}$. Define c_r as the coupon rate of a default-free bond that pays a continuous coupon of $c_r F dt$, matures in τ years, and is issued at a market price equal to its par value, F. Then the fair coupon rate (par yield-to-maturity) for this bond issued at date t equals

$$\begin{aligned}
c_r &= \frac{1 - A\left(\tau \right) e^{-B(\tau) r_t}}{\int_0^\tau A\left(s \right) e^{-B(s) r_t} ds} \\
&\approx \frac{1 - A\left(\tau \right) e^{-B(\tau) r_t}}{\sum_{i=1}^{i=n} A\left(\Delta t \times i \right) e^{-B(\Delta t \times i) r_t} \Delta t}
\end{aligned} \tag{16}$$

where $n = \tau / \Delta t$.

2.2 Credit Spreads on Deposits

Given the risk-neutral distribution of asset returns, it is straightforward to solve for the fair deposit insurance premium or deposit credit risk premium, h_t, as a function of the current asset to deposit ratio, x_t. Since the bank is assumed to be closed by the deposit insurer whenever $x_t \leq 1$, if x_t reaches 1 following a continuous movement in the bank assets, the bank is closed with $A_{t_b} = D_t$ and depositors suffer no loss. Therefore, depositors experience losses only following a downward jump in asset value that exceeds the bank's capital, including contingent

---

[23]Note that this is not the physical process for the interest rate, $dr_t = \kappa^P \left(\bar{r}^P - r_t \right) dt + \sigma_r \sqrt{r_t} d\zeta^P$ where $\kappa = \kappa^P + \psi$ and $\bar{r} = \bar{r}^P \kappa^P / \kappa$ and ψ is a parameter reflecting the price of interest rate risk. Empirical evidence suggests that ψ is negative, so that $\bar{r} > \bar{r}^P$.

capital if it has not yet converted or subordinated debt.[24]   If such a jump does occur at date $\hat{t}$, the instantaneous proportional loss to deposits is $\left(D_t - Y_{q_{\hat{t}-}} A_{\hat{t}-}\right)/D_t$.   At any point in time, the credit risk premium on the instantaneous-maturity deposits, h_t, must reflect the risk-neutral expectation of such a loss.   Thus, the risk-neutral rate of return process for deposits equals

$$\frac{dD^*}{D^*} = (r_t + h_t)\, dt - \max\left(\frac{D_t - Y_{q_{t-}} A_{t-}}{D_t}, 0\right) dq_t \tag{17}$$

For the risk-neutral instantaneous expected return on deposits to equal the risk-free rate, it must be that

$$h_t = \lambda_t E_t^Q \left[\max\left(\frac{D_t - Y_{q_{t-}} A_{t-}}{D_t}, 0\right)\right] \tag{18}$$

To calculate h_t, additional assumptions regarding the risk-neutral frequency of jumps and the distribution of jumps sizes are required.   Specifically, let us assume that $\lambda_t = \lambda$, a constant, and that risk-neutral jump sizes are independent and identically distributed draws from the lognormal distribution:[25]

$$\ln\left(Y_{q_{t-}}\right) \sim N\left(\mu_y, \sigma_y^2\right) \tag{19}$$

and therefore $k_t \equiv E_t^Q[Y_{q_{t-}} - 1] = e^{\mu_y + \frac{1}{2}\sigma_y^2} - 1$ also is a constant.   With these assumptions, the

Appendix shows that

$$h_t = \lambda\left[N\left(-d_1\right) - x_{t-}\exp\left(\mu_y + \frac{1}{2}\sigma_y^2\right)N\left(-d_2\right)\right] \tag{20}$$

where

$$d_1 = \frac{\ln\left(x_{t-}\right) + \mu_y}{\sigma_y}$$
$$d_2 = d_1 + \sigma_y$$

Since h_t changes continuously with the asset-to-deposit ratio, x_t, while the bank remains in operation, depositors always receive fair compensation for their risk of loss and the value of deposits always equals their par value of D_t.

---

[24]The formula for h_t as a function of x_t is unchanged if, for the case of subordinated debt, the regulatory closure threshold is $A_t \leq D_t + B$, rather than $A_t \leq D_t$.   In either case, for any bank currently in operation, a downward jump in asset value is necessary for depositors to suffer a loss.

[25]With little additional complexity, an additional state variable could be introduced to change the jump parameters.   This state variable might be tied to aggregate uncertainty measures, such as the S&P 500 volatility index, VIX.

2.3 Valuing Contingent Capital

Consider a bank that issues deposits, contingent capital, and shareholders' equity. Because deposit credit spreads adjust continuously to fairly compensate depositors for potential losses, the date t sum of contingent capital and original shareholders' equity always equals total capital, $A_t - D_t$, as long as capital is non-negative. As a result, if the value of contingent capital can be derived, the value of original shareholders' equity equals the residual capital. Moreover, any changes in the model's state variables (x_t and r_t) transfers value only between contingent capital investors and shareholders, not depositors.

Recall that the model assumes that contingent capital is issued at date 0 having a value, V_0, equal to its par value, B. Thus, at issue, the contingent capital's fixed-coupon rate, c, or its floating-coupon spread, s, is set such that $V_0 = B$. The equilibrium coupon rate or spread is found by valuing contingent capital for a given coupon rate or spread and then iterating over c or s until one finds the value c^* or s^* such that $V_0 = B$. c^* or s^* will then be the fair coupon rate or spread at the contingent capital's issue date. Accordingly, the date 0 value of original shareholders' equity is simply $E_0 = A_0 - B - D_0$.

Valuing contingent capital for a given coupon rate or spread is carried out using the risk-neutral valuation (martingale pricing) method:

$$V_0 = E_0^Q \left[\int_0^T e^{-\int_0^t r_s ds} v(t) \, dt \right] \tag{21}$$

where $v(t)$ is the contingent capital bond's cashflow per unit time paid at date t. The cashflow equals $c_t B$ as long as the bond is not converted or the bank does not fail, where $c_t = c$ for a fixed-coupon bond and $c_t = r_t + s$ for a floating coupon bond. If at date T the bond has not been converted and the bank has not failed, there is a final cashflow of B. If the bond is converted, say at date t_c, there is the one-time cashflow given by equation (5). Thereafter, $v(t) = 0$ for all $t > t_c$.

Given the bank's initial asset and deposit values, A_0 and D_0, respectively, as well as the initial default-free interest rate, r_0, equation (21) can be computed using the Monte Carlo simulation technique of Boyle (1977). The risk-neutral process followed by the bank's assets equals the assets' risk-neutral rate of return less the payout of interest and premiums to depositors and, as long as contingent capital is unconverted, coupons to contingent capital investors:

$$dA_t = [(r_t - \lambda k)A_t - (r_t + h_t)D_t - c_t B] \, dt + \sigma A_t \, dz + \left(Y_{q_{t^-}} - 1\right) A_t \, dq \tag{22}$$

The asset process in equation (22) can be rewritten as

$$
\begin{aligned}
\frac{dA_t}{A_t} &= \left[(r_t - \lambda k) - (r_t + h_t) \frac{D_t}{A_t} - c_t b_t \frac{D_t}{A_t} \right] dt \, + \, \sigma \, dz \, + \left(Y_{q_{t-}} - 1 \right) dq_t \\
&= \left[(r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} \right] dt \, + \, \sigma \, dz \, + \left(Y_{q_{t-}} - 1 \right) dq_t \qquad (23)
\end{aligned}
$$

Making the change in variables $x_t = A_t/D_t$ and recalling the deposit growth process in equation (4), the risk-neutral process for the asset-to-deposit ratio is

$$
\begin{aligned}
\frac{dx_t}{x_t} &= \frac{dA_t}{A_t} - \frac{dD_t}{D_t} \\
&= \left[(r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g \left(x_t - \widehat{x} \right) \right] dt \, + \, \sigma \, dz \, + \left(Y_{q_{t-}} - 1 \right) dq_t \qquad (24)
\end{aligned}
$$

A simple application of Itô's lemma for jump-diffusion processes implies

$$
d \ln (x_t) = \left[(r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g \left(x_t - \widehat{x} \right) - \frac{1}{2} \sigma^2 \right] dt \, + \, \sigma \, dz \, + \ln Y_{q_{t-}} \, dq_t \qquad (25)
$$

For a given coupon structure, c_t, the risk-neutral processes for the default-free interest rate r_t in equation (12) and the log asset to deposit ratio in equation (25) can be simulated where h_t at each point in time satisfies (20) and $b_t = B/D_t$ evolves as

$$
db_t/b_t = g \left(\widehat{x} - x_t \right) dt. \qquad (26)
$$

By computing the term in brackets in (21) for each simulation and then averaging over a large number of simulations, the contingent capital's initial value, V_0, is determined. The equilibrium coupon rate, c, or coupon spread, s, is found by iterating until $V_0 = B$.

Specifically, solutions for the valuation equation (21) are calculated using a technique similar to Zhou (2001) who provides a discretization method for carrying out a Monte Carlo simulation of a mixed jump-diffusion process. His method is generalized to also consider the stochastic term structure of default-free yields. The time interval $[0, T]$ is divided into n equal sub-periods where $\Delta t \equiv T/n$ is the length of each subperiod. The number n is chosen to be relatively large, making the length of each subperiod, Δt, sufficiently small so that it is a good approximation to assume there can be at most one jump during each subperiod. For example, with time measured in years, then $\Delta t = \frac{1}{250}$ can be set at one trading day. This is the time interval used in the analysis presented in the next section.

Let t denote the end of trading day $t - \Delta t$ and the beginning of trading day t. Then based on equation (12), the change in the default-free interest rate from day t to day $t + \Delta t$ can be

16

approximated as

$$
\begin{aligned}
r_{t+\Delta t} &= r_t + \kappa \left(\bar{r} - r_t \right) \Delta t + \sigma_r \sqrt{r_t} \sqrt{\Delta t} \eta_{t+\Delta t} \\
&= \Delta t \kappa \bar{r} + \left(1 - \Delta t \kappa \right) r_t + \sigma_r \sqrt{r_t \Delta t} \eta_{t+\Delta t}
\end{aligned}
\tag{27}
$$

where $\eta_{t+\Delta t} \sim N\left(0,1\right)$ are serially independent shocks representing Brownian motion uncertainty. Similarly, the daily risk-neutral process for the log of the bank's asset to deposit ratio, equation (25) is approximated as

$$
\begin{aligned}
\ln x_{t+\Delta t} &= \ln x_t + \left[\left(r_t - \lambda_t k_t \right) - \frac{r_t + h_t + cb_t}{x_t} - g\left(x_t - \widehat{x} \right) - \frac{1}{2}\sigma^2 \right] \Delta t \\
&\quad + \sigma\sqrt{\Delta t}\varepsilon_{t+\Delta t} + \ln Y_{t+\Delta t}\varphi_{t+\Delta t}
\end{aligned}
\tag{28}
$$

where $\varepsilon_{t+\Delta t} \sim N\left(0,1\right)$ are serially independent shocks, $E_t^Q\left[\varepsilon_{t+\Delta t}\eta_{t+\Delta t}\right] = \rho$, $\ln\left(Y_{t+\Delta t}\right) \sim N\left(\mu_y, \sigma_y^2\right)$,

$$
\varphi_{t+\Delta t} = \begin{cases} 1 & \text{with probability } \Delta t \lambda_t \\ 0 & \text{with probability } 1 - \Delta t \lambda_t \end{cases},
\tag{29}
$$

h_t is given by (20), and

$$
b_{t+\Delta t} = b_t \exp\left[-g\left(x_t - \widehat{x} \right) \Delta t \right].
\tag{30}
$$

2.4 Valuing Subordinated Debt

If a bank issues subordinated debt rather than contingent capital, the valuation process is similar. Subordinated debt is paid a continuous coupon $c_t B dt$ unless the bank is closed, with closure occurring whenever $x_t \leq 1 + b_t$, where $b_t = B/D_t$ is now the ratio of the subordinated debt's par value to the par value of deposits. As with contingent capital, by varying the fixed coupon rate, c, or the floating coupon spread, s, the initial value of subordinated debt is varied until one finds the coupon rate or spread such that its initial value equals par, B.

2.5 Valuing Contingent Capital with a Dual Price Trigger

Valuing contingent capital with a dual price trigger requires an additional state variable, $\ln I_t$, in the Monte Carlo simulation such that

$$
\ln I_{t+\Delta t} = \ln I_t + \left(r_t - \frac{1}{2}\sigma_i^2 \right) \Delta t + \sigma_i \sqrt{\Delta t}\nu_{t+\Delta t}
\tag{31}
$$

where $\nu_{t+\Delta t} \sim N\left(0,1\right)$ are serially independent shocks that are cross-sectionally correlated with the $\varepsilon_{t+\Delta t}$ and $\eta_{t+\Delta t}$ shocks driving the individual bank's asset returns and the default-free term

structure. Dual price trigger contingent capital is paid a coupon $c_t B dt$ until either conversion occurs or the bank is closed. Assuming $\overline{I}_t = \delta I_0$, where $\delta < 1$, conversion would occur at the first instance when both $I_t \leq \delta I_0$ and $x_t \leq \overline{x}_t$, and its conversion value would equal equation (5). If $I_t > \overline{I}_t$, the bank remains in operation until $x_t \leq 1 + p b_t$, at which time it is closed and the terminal value of contingent capital equals equation (10). If maturity occurs before closure or conversion, the contingent capital bond's terminal value equals $\min[B, A_T - D_T]$.

3   Results

To examine how contract terms affect valuation and the bank's risk-taking incentives, this section computes model values for a given set of benchmark parameters. The parameters of the default-free term structure are similar to those estimated by Duan and Simonato (1999) and equal $\kappa = 0.114$, $\sigma_r = 0.07$, and $\overline{r} = 0.069$. The initial (date 0) instantaneous-maturity interest rate is assumed to be $r_0 = 3.5\%$. These assumptions produce an upward sloping term structure such that the fair default-free coupon (par) rate for a five-year maturity coupon bond given by c_r in equation (16) equals 4.23%.

Ideally, parameters of the bank's asset return jump-diffusion process might be estimated from information on a bank's stock returns, debt prices, and/or credit default swap spreads. Unfortunately, there appears to be no prior research carrying out such an estimation, and performing this exercise is left to future research. The current paper simply assumes what might be reasonable benchmark parameters: $\sigma = 0.02$, $\rho = -0.2$, $\lambda = 1$, $\mu_y = -0.01$, and $\sigma_y = 0.02$. In words, the bank's asset returns have an annual standard deviation deriving from Brownian motion uncertainty of $\sigma = 2\%$. These Brownian motion returns are negatively correlated changes in short-term interest rates with correlation coefficient $\rho = -0.2$.[26] The risk-neutral frequency of jumps, λ, is once per year and the risk-neutral mean jump size is $\mu_y = -1\%$ with a standard deviation of $\sigma_y = 2\%$. If a bank has an equity-to-asset ratio of 10%, these jump-diffusion parameter assumptions produce a standard deviation of bank stock returns of approximately 35%.

3.1   Deposit Credit Spreads

Given the jump process parameters of $\lambda = 1$, $\mu_y = -0.01$, and $\sigma_y = 0.02$ along with a given ratio of assets to deposits, x_t, the fair credit spread h_t can be computed from equation (20). Figure 1 plots the fair credit spread, in basis points, for various capital-to-deposit ratios, $x_t - 1$, ranging from 0.5% to 10%. Schedule A is the credit spread for this benchmark parameter case.

---

[26] This is approximately the long-run daily correlation between Treasury bill yields and the S&P 500.

As expected, the credit spread is inversely related to the capital-to-deposit ratio because lower capital makes it more likely that a downward jump in asset value will wipe out the remaining capital and cause a loss to depositors. Schedule B is the same as Schedule A except that the volatility of jumps, σ_y, is increased from 2% to 3%. It can be seen that more volatile jumps raise credit spreads at each level of capital. Schedule C deviates from the benchmark case by changing the mean jump size, μ_y, from -1% to -2%, and this also leads to higher credit spreads, particularly for low levels of capital. Finally, Schedule D raises the risk-neutral frequency of jumps, λ, from once per year to twice per year. As can be seen from equation (20), this simply doubles the benchmark case credit spread for each level of capital.

3.2 Yields on Contingent Capital

This section presents fair, new issue yields for fixed-coupon contingent capital as well as fair new issue spreads for floating-rate contingent capital. In addition to the benchmark parameters described earlier, it is assumed that the bank has a target capital to deposit ratio of 10%; that is, $\hat{x} = 1.10$. Moreover, the mean-reversion parameter for bank deposit growth is $g = 0.5$, implying that when the bank's capital ratio deviates from target, the expected reduction in the deviation over the next year is approximately one half.

The benchmark contingent capital bond is assumed to have a five-year maturity ($\tau = 5$) and a new issue amount (par value) equal to 4% of deposits ($b_0 = 0.04$). Thus, if the bank is initially at its 10% target capital ratio, 4% is contingent capital and 6% is original shareholders' equity. This benchmark bond is specified to convert at par ($p = 1$) when the market value of original shareholders' equity falls to 2% of deposits; that is, $\bar{e} = 2\%$. Hence, using the conversion threshold rule discussed earlier of $\bar{x}_{t_h} = 1 + \bar{e} + pb_{t_h}$, conversion of this benchmark bond will tend to occur when total capital is approximately 6% or less of deposits.

3.2.1 Jumps and Mean-Reversion of Capital Ratios

Figure 2 gives the new issue yields for fixed-coupon contingent capital, c, when the bank's initial total capital ranges from 6.5% to 15%. Recall that the default-free term structure is assumed to have an initial instantaneous maturity interest rate of r_0 equal to 3.5% and the par yield on a five-year Treasury coupon bond is 4.23%. This 4.23% default-free, five-year par yield is given by the dashed line denoted Schedule A in the figure. In comparison, Schedule B of Figure 2 shows that the benchmark contingent capital bond's new issue yield is 5.41%, 4.56%, and 4.39% when initial capital is 6.5%, 10%, and 15%, respectively.

This contingent capital bond's yield spread above the five-year Treasury is due to the possibility that it could convert at less than par following a downward jump in the bank's asset (and

equity) value. If all of the benchmark parameters are maintained except one assumes there is no possibility of jumps ($\lambda = 0$), then the contingent capital bond's spreads over the five-year Treasury yield would not be positive. Indeed, given the assumption of an upward-sloping term structure, Schedule C of Figure 2 shows that spreads would be slightly negative. Since conversion lowers the effective maturity of contingent capital and, without jumps, it always converts at par, it is effectively a default-free bond with a maturity of less than five years. Hence, its yield is more like a that of a shorter-term default-free bond, which is below the five-year default-free yield. Thus, one sees that the possibility of jumps in the bank's asset value, as might occur during a financial crisis, has a qualitatively important impact on the pricing of contingent capital.

Schedule D of Figure 2 maintains the benchmark bond's parameters except that the mean-reversion parameter for bank deposit growth is lowered from $g = 0.5$ to $g = 0.25$. Such a bank is slower to adjust deposits in order to move toward its target capital to deposit ratio of 10%. The effect is to raise new issue yields when the bank has low capital but lower them when the bank has high capital. The intuition for this result is that if the bank starts out undercapitalized, slower capital ratio reversion tends to keep it undercapitalized for a longer time, thereby increasing opportunities where a downward jump in asset value could require conversion at less than par. In contrast, if the bank starts out overcapitalized, slower capital ratio reversion tends to keep it overcapitalized for a longer time, reducing the likelihood that a downward jump in asset value could require conversion at less than par.

3.2.2 Maturity

Figure 3 examines how new issue yields for fixed-coupon contingent capital vary by maturity. The dashed-line Schedules A, B, and C give the default-free par coupon rates for 3-, 5-, and 10-year Treasury bonds, which are 3.99%, 4.23%, and 4.64%, respectively. Schedules D, E, and F then show the new issue yields for contingent capital having the benchmark parameters except that their times until maturity are 3 years, 5 years, and 10 years, respectively.[27] When the bank has high capital, the yields on contingent capital bonds approach the default-free yields for their respective maturities. However, when capital is low, the contingent capital bonds have more similar yields because with conversion a significant possibility, they are valued more as shareholders' equity than as default-free bonds. When capital is 7.5% of deposits or less, their yields converge in the 5% to 5.6% range, due to having similar high probabilities of experiencing a downward jump in asset value that could require conversion at less than par. Note that when capital is low and the likelihood of conversion losses are high, the contingent capital bonds' spreads over their respective default-free Treasury yields are a decreasing function of maturity,

---

[27]Schedules B and E for the benchmark five-year maturity are the same as those in Figure 2.

a result consistent with other structural models, such as Merton (1974).[28]

3.2.3 Conversion Terms

Figure 4 considers how a contingent capital bond's conversion features affect its new issue yields. Schedules A and B are repeated from Figure 2 and are the five-year default-free par yield and the par yield on the benchmark contingent capital bond where $p = 1$ and $\bar{e} = 2\%$. Schedule C gives the new issue par yield for five-year contingent capital having conversion specified to equal 90% of par value; that is $p = 0.9$ and $\bar{e} = 2.4\%$.[29] Notably, the fair coupon rate for contingent capital that converts at a discount is significantly higher than the benchmark case, particularly for bonds issued when bank capital is low. The higher yield due to the conversion discount is not dependent on asset value jumps. Because the conversion discount imposes losses on bondholders, yields would be above the default-free yield even if bank assets followed a pure diffusion process.

Also in Figure 4, a five-year contingent capital bond that converts at full value ($p = 1$) but at a smaller equity threshold of $\bar{e} = 1\%$ is given in Schedule D. In this case, conversion occurs at or below a total capital ratio threshold of 5%, so new issue yields are graphed over the capital to deposit ratios of 5.5% to 15%. Importantly, this contingent capital bond's yields are higher than the benchmark $p = 1$, $\bar{e} = 2\%$ case because the smaller 1% equity cushion makes it more likely that a downward jump in asset value can occur that would prevent full conversion. In other words, for the benchmark contingent capital bond, at a point just before conversion, there would need to be a sudden asset value loss exceeding 2% to prevent full conversion, while for the contingent capital bond with $\bar{e} = 1\%$, at a point just before conversion, there would need to be a sudden asset value loss only slightly more than 1% for bondholders to sustain a conversion loss. This finding has implications for recent regulatory proposals that would have contingent capital convert only when a bank was in dire straits and close to being seized by regulators.[30] Delaying conversion to a point when the value of original shareholders' equity is low raises the new issue yields on contingent capital.

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[28] For example, at 6.5% capital, the contingent capital bond spreads over equivalent maturity default-free yields are 162, 117, and 80 basis points for maturities of 3, 5, and 10 years. The inverse spread maturity relationship holds for bonds with relatively high default risk. In contrast, Merton (1974) finds a hump-shaped yield-maturity relationship for bonds with relatively low default risk. This is also true in our model's example, since at 15% capital, the contingent capital bonds' spreads over equivalent maturity default-free yields are 15, 16, and 9 basis points for maturities of 3, 5, and 10 years, respectively.

[29] Recall that if contingent capital converts at a discount to face value, the resulting total capital will be less than if the conversion was at face value. To correct for this, use equation (8) to raise $\bar{e}$ to be higher for the case when contingent capital converts at discount relative to the no-discount case; that is, if $\bar{e}_{p=1} = 2\%$, then $\bar{e}_{p=0.9} = 2.4\%$. Making this adjustment, the conversion threshold would stay at the point were deposits are 1.06% of assets.

[30] Canada's superintendent of financial institutions, Julie Dickson, proposes that the conversion trigger for contingent capital would be "when the regulator is ready to seize control of the institution because problems are so deep that no private buyer would be willing to acquire shares in the bank." *Financial Times*, April 9, 2010.

The final Schedule E in Figure 3 gives the new issue yields on fixed-coupon, five-year subordinated debt having an initial par value equal to 4% of deposits. Recall that it is assumed when a bank issues subordinated debt, that regulators close the bank when assets fall to, or below, the total of the par value of deposits plus subordinated debt. Thus, subordinated debt can be viewed as similar to contingent capital with $p = 1$ but $\overline{e} = 0$; that is, subordinated debt has no equity conversion buffer. This makes it more likely that a downward jump in asset value could impose losses and explains why subordinated debt's yields are higher than contingent capital that converts at par (but not as high as contingent capital that converts at a discount).

Figure 5 performs similar analysis to that of Figure 4 but for floating-rate contingent capital. It graphs s, the new issue credit spread (over the instantaneous maturity interest rate) for contingent capital bonds with different conversion terms. Note that a zero credit spread represents no default risk, and this is the equilibrium credit spread for all contingent capital that convert at par ($p = 1$) if there were no possibility of jumps in asset returns. Hence, as with the case of fixed-coupon contingent capital, positive spreads on floating-rate, par conversion, contingent capital occur due to the possibility of sudden asset value losses that would prevent full conversion. For example, for the benchmark contingent capital bond of $p = 1$ and $\overline{e} = 2\%$ given in Schedule A, the new issue floating rate spreads (over the instantaneous maturity rate) are 141, 45, and 23 basis points for initial bank capital of 6.5%, 10%, and 15%, respectively.

Figure 5 compares new issue spreads for floating-coupon contingent capital having different conversion features, and its results are nearly identical to those given for fixed-coupon yields in Figure 4. One sees that a smaller equity conversion threshold ($p = 1$, $\overline{e} = 1\%$) raises spreads as does a conversion discount ($p = 0.9$ and $\overline{e} = 2.4\%$). Also, floating-coupon subordinated debt spreads are higher than contingent capital bonds except when conversion occurs at a discount.

3.2.4 After-Issue Values of Contingent Capital and Shareholders' Equity

Thus far, the results have compared new issue yields and spreads. Now let us consider the pricing of floating-coupon contingent capital and shareholders' equity after the bonds are issued, so that spreads are fixed prior to examining subsequent changes in capital.[31] Figure 6 considers the prices of contingent capital bonds that were issued at fair spreads when the bank's capital to deposit ratio equaled 10%, and then examines how prices change as the capital ratio declines. The different contingent capital bonds are the same ones considered earlier: Bond A converts at par when equity is 2% of deposits ($p = 1$, $\overline{e} = 2\%$); Bond B converts at a 10% discount to par when equity is 2.4% of deposits ($p = 0.9$ and $\overline{e} = 2.4\%$); and Bond C converts at par when equity is 1% of deposits ($p = 1$, $\overline{e} = 1\%$). As can be seen from the figure, all three of the bonds equal

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[31] These valuations are done for floating-coupon contingent capital, but the results for fixed-coupon contingent capital are nearly identical.

their par value of 4% of deposits when total capital equals 10% of deposits. As the capital ratio declines, the values of the three bonds tend to fall. However, at a capital ratio of 6.75%, Bond A reaches its minimum value of 3.94%, while at a capital ratio of 5.75 %, Bond C researches its minimum value of 3.85%. The values of these bonds then turn upward as each comes close to its conversion threshold, which is 6.00% capital for Bond A and 5.00% capital for Bond C. The intuition for this upturn in price is that it becomes relatively more likely that the bonds will convert at par compared to below par. While the risk of a downward jump in asset value that would prevent full conversion increases as capital declines, as capital approaches the threshold, the likelihood of hitting the threshold via a continuous Brownian motion decline increases even more quickly. If the threshold is hit in such a continuous manner, then the bond converts at its par value of 4%, so that the price rises toward that level. Bond C, which converts when equity is only 1% at the threshold, never rises as high as Bond A which converts when equity equals 2% at the threshold. This is because Bond C is exposed to a greater likelihood that a sudden decline in asset value would prevent conversion at par.

Price dynamics are qualitatively different for Figure 6's Bond B which converts at a discount to par value. This bond's value declines at an increasing rate as the 6.00% capital ratio threshold is approached. Conversion at the threshold for this bond would be at $0.90 \times 0.04\% = 3.6\%$ of deposits, so that even a continuous decline in asset value would impose losses on the bondholders.

Recall that because credit spreads on deposits adjust instantaneously to the current level of capital, deposits are always priced at par as long as capital is non-negative. Consequently, the sum of the values of contingent capital and original shareholders' equity must equal total capital, $A_t - D_t$. Therefore, subtracting the values of contingent capital bonds in Figure 6 from total capital gives the corresponding equilibrium values of shareholders' equity. These shareholders' equity values are graphed in Figure 7. Consistent with par conversion Bonds A and C having slight upward rises in value as total capital declines to their respective 6% and 5% capital ratio thresholds, the corresponding market values of equity in Schedules A and C decline at slightly greater than one-for-one as capital approaches these thresholds. In contrast, because contingent capital Bond B converts at a discount to par, making its value decline as capital approaches its 6.00% conversion threshold, the corresponding value of shareholders' equity declines at a somewhat less than one-for-one rate as the conversion threshold is met. However, for each Schedule A, B, and C, the equilibrium value of shareholders' equity declines monotonically with a fall in total capital and, from equation (6), their prices approach their full conversion values of $E_{t_c} = A_{t_c} - D_{t_c} - pB$, equal to 2% of deposits, 2.4% of deposits, and 1% of deposits, respectively.

3.2.5 Dual Price Conversion Trigger

Let us now consider the effects of one additional conversion feature, namely, the dual price conversion trigger proposed by McDonald (2009). It is assumed that the financial stock index, I_t, must fall at least 10% from its level at the time that contingent capital is issued; that is, $\overline{I}_t = \delta I_0 = 0.9 I_0$. Similar to McDonald (2009), the volatility of the index's return is assumed to be $\sigma_i = 20\%$, and the index return's correlations with interest rate changes and the bank's asset return are $dz_i d\zeta = -0.2 dt$ and $dz_i dz = 0.85 dt$. Figure 8 compares new issue yields on fixed-coupon contingent capital with, and without, the financial index trigger. As before, Schedule A's dashed line gives the par yield on a five-year Treasury bond while Schedule B repeats the fixed-coupon yields for the benchmark five-year, single trigger contingent capital bond ($p = 1$, $\overline{e} = 2\%$). Schedule C is then the equivalent par-conversion contingent capital bond ($p = 1$, $\overline{e} = 2\%$) except that it has the dual price trigger. As can be seen, its new issue yields are above those of the standard single-trigger contingent capital. However, they are below the new issue yields of subordinated debt graphed in Schedule D.

The logic behind this ordering of yields relates to the previously discussed benefit of an equity cushion. Yields on standard single-trigger contingent capital (Schedule B) are lowest because it is converted at par without loss to its holders when equity hits the 2% threshold. The yields on subordinated debt (Schedule D) are highest because it completely lacks this equity conversion cushion. Dual trigger, par-conversion, contingent capital is an intermediate case because sometimes the equity cushion is effective in providing the protection resulting from par conversion (when $I_t \leq \overline{I}_t$), but other times it is not (when $I_t > \overline{I}_t$). Thus, in some states of the world, dual price trigger contingent capital acts like single price trigger contingent capital, but in other states it acts like non-convertible subordinated debt. Hence, its initial pricing reflects a mix of both convertible and non-convertible debt.

Schedule E of Figure 8 repeats Figure 4's new issue yields of single price trigger, fixed-coupon contingent capital that converts at a discount ($p = 0.9$, $\overline{e} = 2.4\%$). Schedule F of Figure 8 then gives the equivalent contingent capital but with a dual price trigger. Interestingly, for contingent capital that converts at a discount, the impact of the dual price trigger is to lower, rather than raise, yields. However, this should be expected because a discount from par now makes conversion a costly feature for contingent capital investors. As in the par conversion case, when conversion is at a discount the yields on contingent capital with a dual price trigger fall between those of single price trigger contingent capital (Schedule E) and non-convertible subordinated debt (Schedule D). In summary, one can understand the characteristics of dual price trigger contingent capital by viewing it as a blend of standard, single price trigger contingent capital and non-convertible subordinated debt.

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3.3 Incentives for Risk-Taking

This section considers the risk-taking incentives of a bank that issues contingent capital by investigating how changes in asset risk and capital ratios affect the relative values of contingent capital and shareholders' equity. Because credit spreads on short-maturity deposits adjust instantly, changes in the bank's risk does not affect their values. Hence, in the model if a bank issues only deposits and shareholders' equity, it would have no incentive or ability to transfer value from depositors to shareholders by increasing risk. Shareholders' equity would always equal the bank's total capital as long as capital is non-negative. While this model implication is stark, it helps to isolate the incentives of bank shareholders to increase risk for the purpose of exploiting contingent capital investors.

Unlike a structural credit risk model such as Merton (1974) where assets follow a pure diffusion process and asset risk can be summarized by a single parameter, σ, the current paper's model has several additional risk parameters that need to be considered: the risk-neutral probability of jumps (λ); the jump size volatility (σ_y); and the mean jump size (μ_y). As will be seen, these parameters of the risk-neutral distribution of asset returns can have disparate effects on risk-taking incentives.

The analysis of risk-taking incentives considers different forms of contingent capital and subordinated bonds, but where each five-year maturity bond was issued at its fair credit spread when the bond had a par value of 4% of deposits and total bank capital was 10% of deposits.[32] It is assumed that the newly-issued bonds' credit spreads reflect the benchmark asset risk parameter values ($\lambda = 1$, $\sigma_y = 0.02$, $\mu_y = -0.01$, and $\sigma = 0.02$). Then, for a given capital ratio, the market value of original shareholders' equity is computed for a 25% change in the value of one of the asset risk parameters ($\lambda = 1.25$ or $\sigma_y = 0.025$ or $\mu_y = -0.0125$ or $\sigma = 0.025$).[33][34] The change in the market value of shareholders' equity due to this 25% parameter change is graphed in Figures 9 to 12. Note that since the values of original shareholders' equity plus contingent capital or subordinated bonds always sum to total capital, the change in the value of the bond exactly equals minus the change in shareholders' equity.

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[32]Contingent capital and subordinated debt are assumed to pay floating coupons. The results for fixed-coupon bonds are extremely similar.

[33]Admittedly, this parameter change is an out-of-equilibrium event in that it was not foreseen by bondholders when initial credit spreads were set. However, it would be straightforward to model parameter change dynamics in a rational framework. For example, risk parameters might be specified as a function of the bank's asset-to-deposit ratio, x_t, and initial fair credit spreads could be computed via a similar Monte Carlo valuation but where risk parameters vary with the state variable, x_t. Most likely initial credit spreads would rise to reflect this moral hazard but the qualitative results regarding banks' incentives to shift risk would be similar to the current analysis.

[34]A change in the asset risk-parameters does not affect the risk-neutral expected rate of return on the bank's assets, which continues to equal the instantaneous-maturity interest rate, r_t.

3.3.1 Jump Risk

Figure 9 considers the increase in shareholders' equity when there is a 25% increase in the probability of jumps, λ. While in all cases the resulting increase in shareholders' equity (*bond value*) is positive (*negative*), the increase is smallest for the contingent capital Bond A which converts at par with a 2% equity threshold ($p = 1$, $\bar{e} = 2\%$). For moderate and high levels of capital, a rise in the frequency of jumps has a significantly adverse effect on contingent capital Bond B that converts at a discount ($p = 0.9$, $\bar{e} = 2.4\%$). Intuitively, when capital is high, a greater probability of jumps has a larger marginal effect on reducing the value of contingent capital that would suffer a loss at conversion. For contingent capital Bond C that converts when total capital equals 5% ($p = 1$, $\bar{e} = 1\%$), a rise in the probability of jumps is especially damaging at low capital levels because, with its smaller 1% equity conversion cushion, a jump has a greater likelihood of preventing full conversion. Similar reasoning explains why the non-convertible subordinated Bond D creates the greatest incentive for risk shifting: with no conversion buffer, a jump in asset value is more likely to impose losses to subordinated debt investors, thereby transferring more value to shareholders.

Figure 10 presents similar analysis for the case of a 25% increase in the volatility of jumps, σ_y. It is similar to the results in Figure 9 in that risk shifting incentives are greatest when the bank issues subordinated Bond D, followed by contingent capital Bond C which converts at a 5% capital threshold ($p = 1$, $\bar{e} = 1\%$), followed by contingent capital Bond A ($p = 1$, $\bar{e} = 2\%$) and B ($p = 0.9$, $\bar{e} = 2.4\%$) which convert at a 6% capital ratio. This ordering confirms the importance of the conversion threshold in protecting bondholders. A larger capital buffer between the conversion threshold and the bond's par value (0% for subordinated Bond D, 1% for Bond C, and 2% for Bonds A and B) protects bondholders because a sudden loss in asset value that moves capital into this buffer would not harm bondholders. One interesting aspect of the results is that for all four bonds, the incentive for risk taking peaks at capital levels from around 1.5% to 2% above the bond's respective conversion thresholds. A likely explanation is that the calculations measure the marginal effect of an increase in jump volatility on the values of shareholders' equity and bonds. Since an increase in σ_y fattens the tails of the asset return distribution, the marginal effect of a greater tail probability in exposing bondholders to partial conversion losses may be greatest at a point significantly above the capital conversion threshold.

The results in Figure 11 are qualitatively similar to those in Figures 9 and 10. It shows results for a 25% change in the mean jump size, μ_y, from -1% to -1.25%. As with the other jump risk parameters, a bank's incentive to risk-shift is greatest with subordinated Bond D, followed by contingent capital Bond C that converts at the 5% capital threshold. Risk-shifting incentives are lowest for contingent capital Bond A ($p = 1$, $\bar{e} = 2\%$), except very near the capital conversion threshold where the marginal effect for Bond B ($p = 0.9$, $\bar{e} = 2.4\%$) becomes least. Again, these results highlight the critical role of the capital conversion buffer in protecting bondholders from

jump risk.

3.3.2 Diffusion Risk

Finally, Figure 12 calculates the change in the value of shareholders' equity from a 25% increase in the bank asset diffusion volatility, σ. In some ways, the results are qualitatively different from those relating to the jump risk parameters. Except for a bank that issues contingent capital Bond B which converts at a discount ($p = 0.9$, $\bar{e} = 2.4\%$), shareholders have a disincentive to increase diffusion volatility when capital falls near a bond's conversion threshold. The explanation for this finding is that a larger impact of Brownian motion uncertainty makes it more likely that a bond's capital conversion threshold will be reached via a continuous decline in the bank's asset value, rather than a downward jump that could breach the threshold. With a greater likelihood of full conversion occurring at the threshold, there is a smaller possibility of bondholders suffering a loss.[35] Hence, shareholders cannot gain when the bank increases such "small scale" diffusion risk. Contingent capital Bond B is a notable exception because its conversion discount implies that bondholders suffer a loss even when conversion occurs exactly at the threshold.

4 Conclusion

This paper's structural credit risk model provides a framework for valuing contingent bank capital and bank shareholders' equity. The model incorporates a realistic feature of bank asset returns, namely, that they sometimes experience sudden, discrete declines, often during a financial crisis. Since a primary motivation for contingent capital is to alleviate financial distress and protect taxpayers during a crisis, understanding the role of jump risk is critical. Indeed, the possibility of sudden large losses in a bank's asset value has a qualitatively distinct impact on contingent capital credit spreads. Without asset jump risk, standard contingent capital that converts at par would be default-free and require a zero credit spread. With asset jump risk, conversion at below par value becomes feasible, so that new-issue credit spreads for contingent capital become positive.

Credit spreads for both fixed- and floating-coupon contingent capital will be higher when they are issued at low levels of bank capital and when the conversion threshold of original shareholders' equity is low. Contingent capital investors will require higher new-issue credit spreads, even in the absence of jump risk, if the conversion terms specify a discount to par value since losses occur at conversion. The effect of a dual price trigger for conversion is to make

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[35]This reasoning holds even for subordinated debt investors due to the assumption that the bank is closed when total capital falls to equal the par value of subordinated debt.

contingent capital a blend of non-convertible subordinated debt and standard single price trigger contingent capital. Therefore, yields on dual price trigger contingent capital will fall between those of comparable single price trigger contingent capital and subordinated debt.

A bank that issues contingent capital faces a moral hazard incentive to increase its assets' jump risks. However, this incentive to transfer value from contingent capital investors to the bank's shareholders is smaller than that when the bank has issued a similar amount of subordinated debt rather than contingent capital. Thus, relative to the status quo, there is likely to be a decline in moral hazard if contingent capital replaces subordinated debt. The results show that excessive risk-taking incentives also decline as contingent capital's equity conversion threshold rises. With a bigger "equity cushion" at the conversion threshold, there is a smaller likelihood that a sudden loss in bank asset value would prevent full conversion, thereby better protecting contingent capital investors from losses.

In conclusion, this paper's structural analysis suggests that contingent capital would be a feasible, low-cost method of mitigating financial distress, particularly when its conversion threshold is set at a relatively high level of original shareholders' equity. Indeed, contingent capital may reduce a bank's moral hazard incentives relative to other forms of debt-like capital. Because it reduces effective leverage and the pressure for government bailouts, contingent capital deserves serious consideration as part of a package of reforms that stabilize the financial system and eliminate "Too-Big-to-Fail."

Appendix

This appendix derives the formula for h_t in equation (20).

Define

$$
\begin{aligned}
H &\equiv E_t^Q \left[\max \left(\frac{D_t - Y_{q_{t-}} A_{t-}}{D_t}, 0 \right) \right] \\
&= E_t^Q \left[\max \left(1 - Y_{t-} x_{t-}, 0 \right) \right] \\
&= \int_0^{1/x} (1 - Yx) \exp \left[-\frac{(\ln Y - \mu_y)^2}{2\sigma_y^2} \right] \frac{1}{Y \sigma_y \sqrt{2\pi}} dY.
\end{aligned}
\tag{A.1}
$$

Make the change of variable

$$
y = \frac{\ln Y - \mu_y}{\sigma_y};
$$

then $y\big|_{Y=0} = -\infty$, $y\big|_{Y=1/x} = \frac{\ln 1/x - \mu_y}{\sigma_y} = -\frac{\ln x + \mu_y}{\sigma_y}$, $Y = \exp \left[\mu_y + y\sigma_y \right]$, and $dy = \frac{1}{Y\sigma_y} dY$.
Defining

$$
d_1 \equiv \frac{\ln x + \mu_y}{\sigma_y},
$$

then

$$
H = \int_{-\infty}^{-d_1} \left(1 - \exp \left[\mu_y + y\sigma_y \right] x \right) \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy = N(-d_1) - x e^{\mu_y} \int_{-\infty}^{-d_1} \exp \left[y\sigma_y - \frac{y^2}{2} \right] \frac{1}{\sqrt{2\pi}} dy.
\tag{A.2}
$$

Completing the square in the exponent, one obtains

$$
\int_{-\infty}^{-d_1} \exp \left[y\sigma_y - \frac{y^2}{2} \right] \frac{1}{\sqrt{2\pi}} dy = e^{\sigma_y^2/2} \int_{-\infty}^{-d_1} \frac{\exp \left[-\frac{1}{2}(y - \sigma_y)^2 \right]}{\sqrt{2\pi}} dy = e^{\sigma_y^2/2} \int_{-\infty}^{-d_2} \frac{\exp \left[-\frac{y^2}{2} \right]}{\sqrt{2\pi}} dy.
\tag{A.3}
$$

where

$$
d_2 = d_1 + \sigma_y = \frac{\ln x + \mu_y}{\sigma_y} + \sigma_y.
$$

Collecting things together, one finds

$$
H = N(-d_1) - x \exp \left[\mu_y + \frac{\sigma_y^2}{2} \right] N(-d_2) = N(-d_1) - \exp \left[\ln x + \mu_y + \frac{\sigma_y^2}{2} \right] N(-d_2). \tag{A.4}
$$

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Figure 1

Deposit Credit Spreads (h_t)
(in basis points)

A. Benchmark $\lambda = 1$, $\mu_y = -1\%$, $\sigma_y = 2\%$

B. $\sigma_y = 3\%$

C. $\mu_y = -2\%$

D. $\lambda = 2$
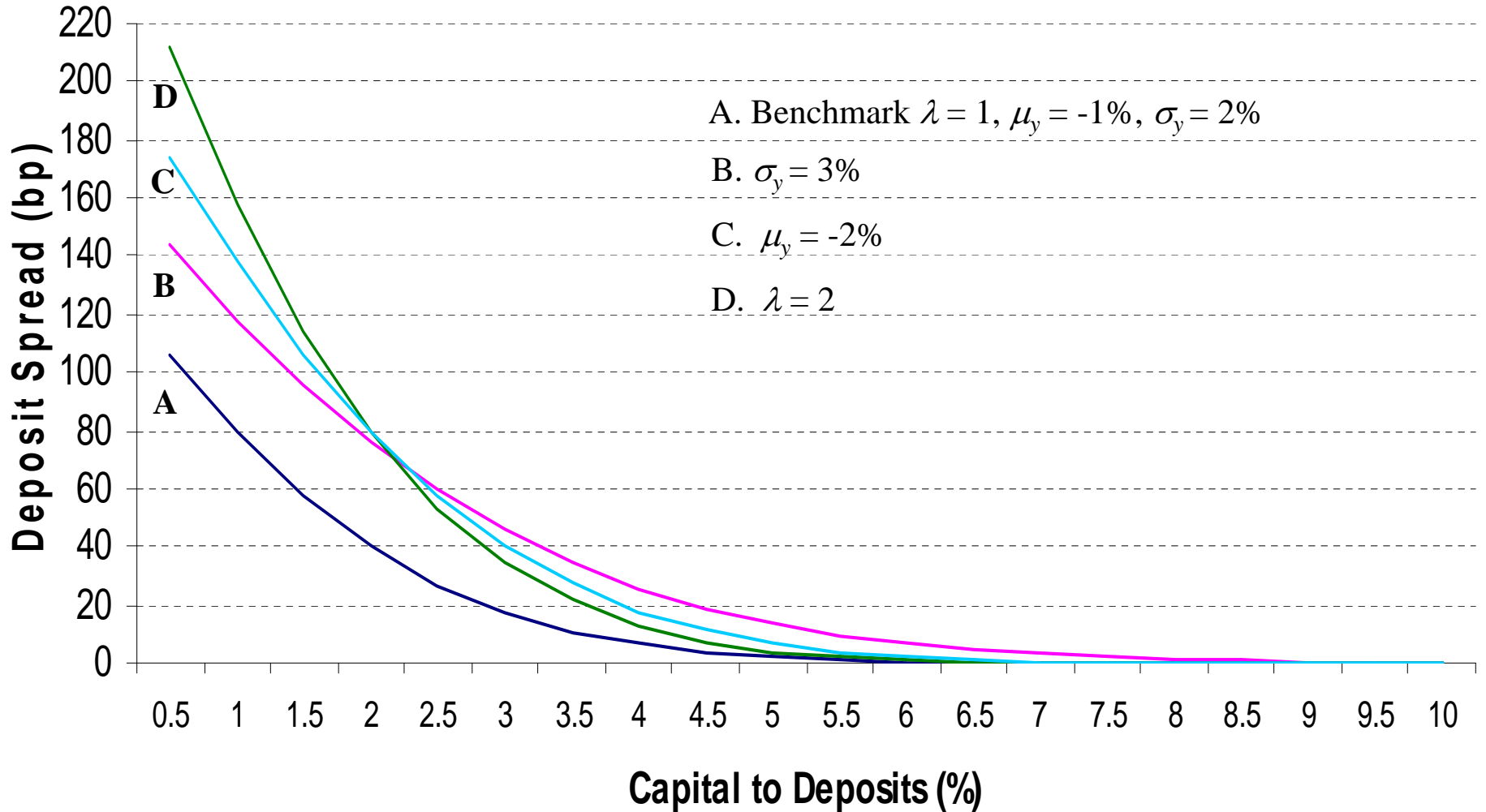
Figure 2

**New Issue Par Yields on Fixed-Coupon Contingent Capital (c)
Effects of Jumps in Asset Values and Mean-Reverting Leverage**
(Five-Year Maturity, Initial Value = 4% of Deposits)

A. Default-Free Five-Year Coupon Bond
B. Contingent Capital: $p = 1$, $\bar{e} = 2\%$, $\lambda = 1$, $g = 0.5$
C. Contingent Capital with No Jumps: $p = 1$, $\bar{e} = 2\%$, $\lambda = 0$, $g = 0.5$
D. Contingent Capital with Slower Reversion: $p = 1$, $\bar{e} = 2\%$, $\lambda = 1$, $g = 0.25$

Coupon Rate (%)

Capital to Deposits (%)

33
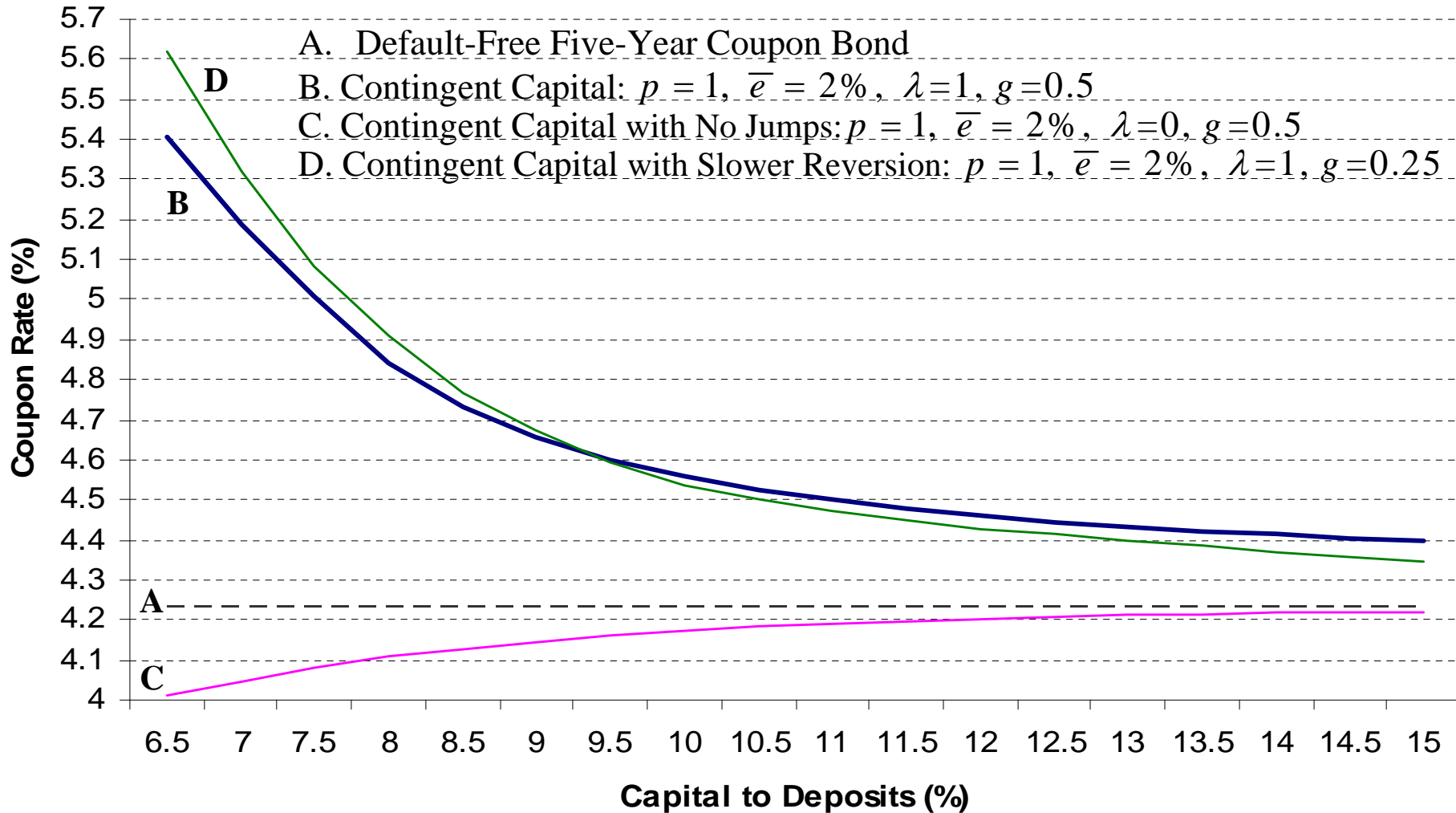
Figure 3

New Issue Par Yields on Fixed-Coupon Contingent Capital (*c*)
Effects of Maturity
(Initial Value = 4% of Deposits)

A. Default-Free Three-Year Coupon Bond
B. Default-Free Five-Year Coupon Bond
C. Default-Free Ten-year Coupon Bond
D. Contingent Capital: T = Three Years
E. Contingent Capital: T = Five Years
F. Contingent Capital: T = Ten Years

Coupon Rate (%) (y-axis)

Capital to Deposits (%) (x-axis)

Figure 4

New Issue Par Yields on Fixed-Coupon Contingent Capital (c)
Effects of Conversion Terms
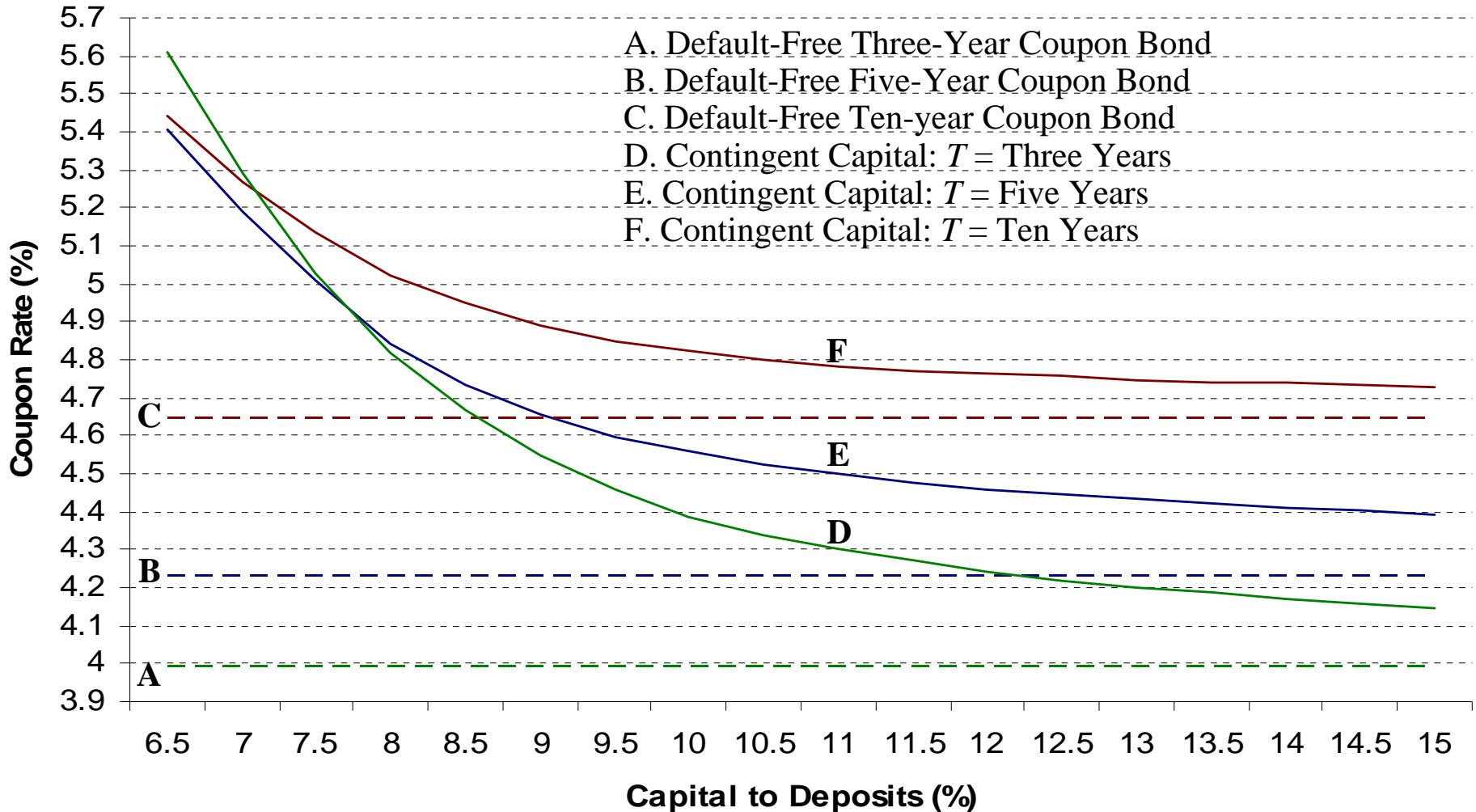### (Five-Year Maturity, Initial Value = 4% of Deposits)

A. Default-Free Five-Year Coupon Bond
B. Contingent Capital: $p = 1,\ \overline{e} = 2\%$
C. Contingent Capital: $p = 0.9,\ \overline{e} = 2.4\%$
D. Contingent Capital: $p = 1,\ \overline{e} = 1\%$
E. Subordinated Debt

Coupon Rate (%)

Capital to Deposits (%)

35

Figure 5

New Issue Credit Spreads on Floating-Rate Contingent Capital (*s*)
Effects of Conversion Parameters
(Five-Year Maturity, Initial Value = 4% of Deposits)

Legend:
A. Contingent Capital: $p = 1$, $\overline{e} = 2\%$
B. Contingent Capital: $p = 0.9$, $\overline{e} = 2.4\%$
C. Contingent Capital: $p = 1$, $\overline{e} = 1\%$
D. Subordinated Debt

Y-axis: Credit Spread (bp)
X-axis: Capital to Deposits (%)

36
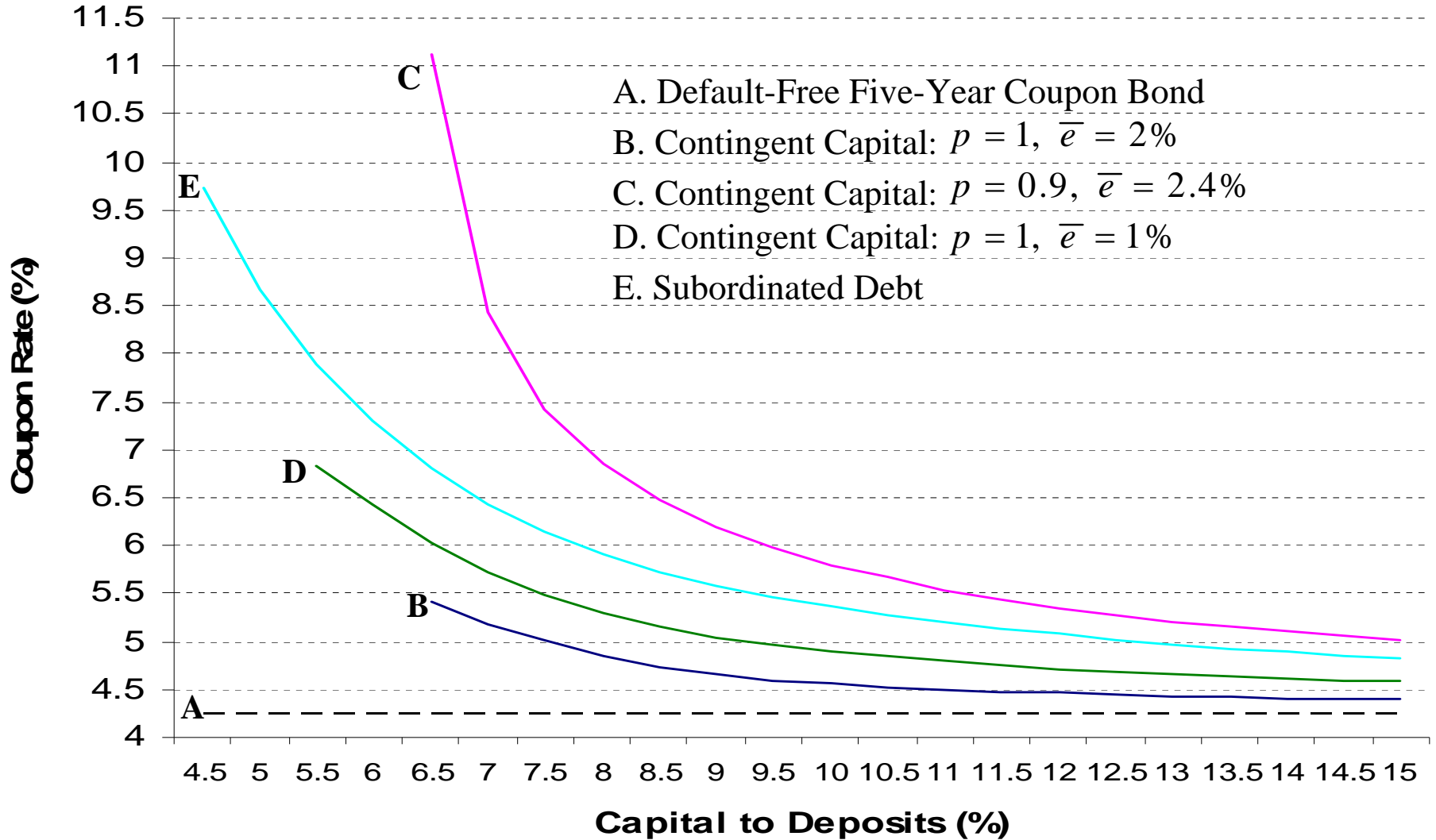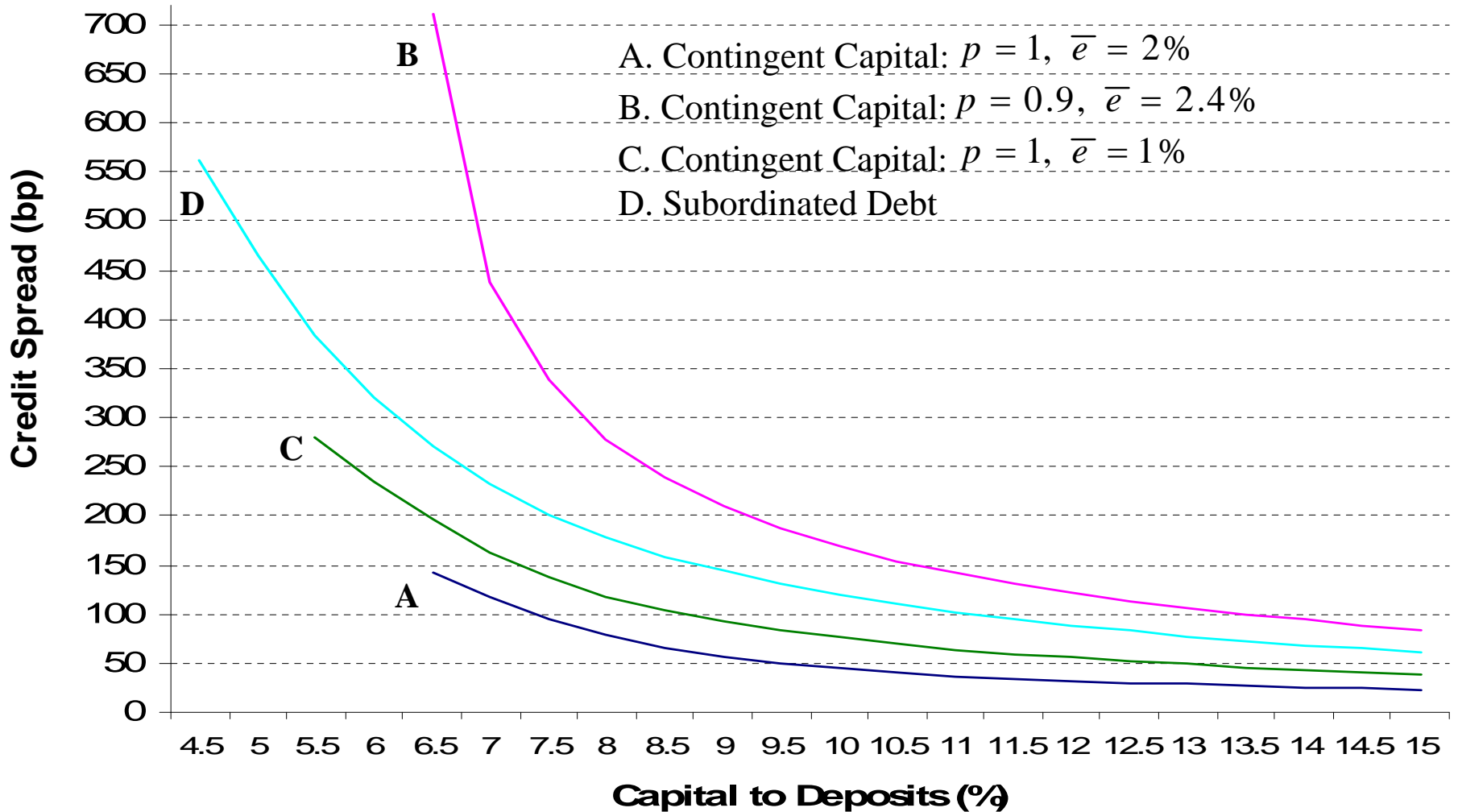
Figure 6

Value of Floating-Rate Contingent Capital
Effects of Conversion Parameters
(Five-Year Maturity, Initial Value = 4% of Deposits)

A. Contingent Capital: $p = 1$, $\overline{e} = 2\%$

B. Contingent Capital: $p = 0.9$, $\overline{e} = 2.4\%$

C. Contingent Capital: $p = 1$, $\overline{e} = 1\%$

37
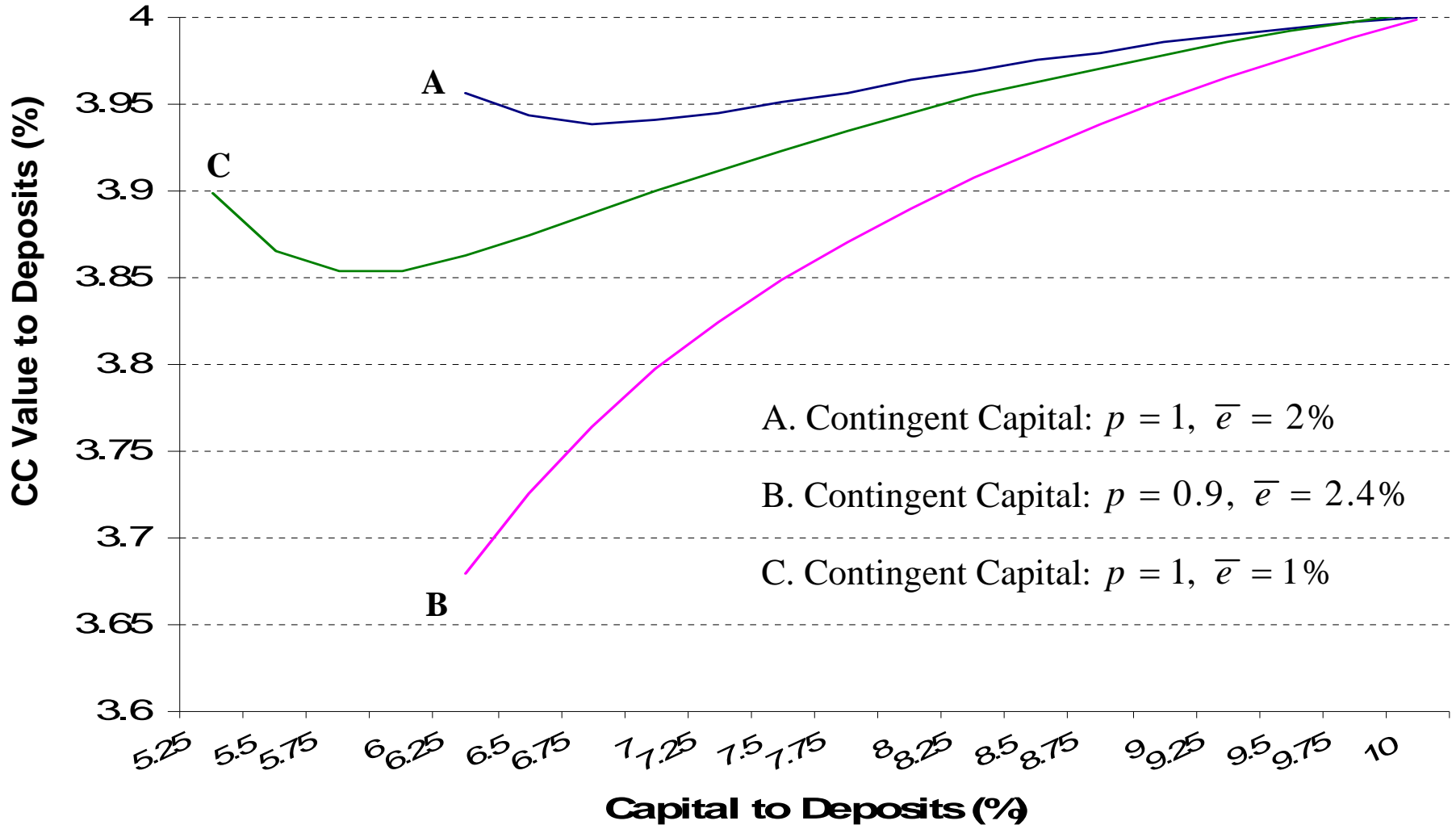
Figure 7

Value of Shareholders' Equity with Floating-Rate Contingent Capital Effects of Conversion Parameters

(Five-Year Maturity, Initial Value = 4% of Deposits)

A. Equity with Contingent Capital: $p = 1, \ \overline{e} = 2\%$

B. Equity with Contingent Capital: $p = 0.9, \ \overline{e} = 2.4\%$

C. Equity with Contingent Capital: $p = 1, \ \overline{e} = 1\%$

Equity Value to Deposits (%) (y-axis)

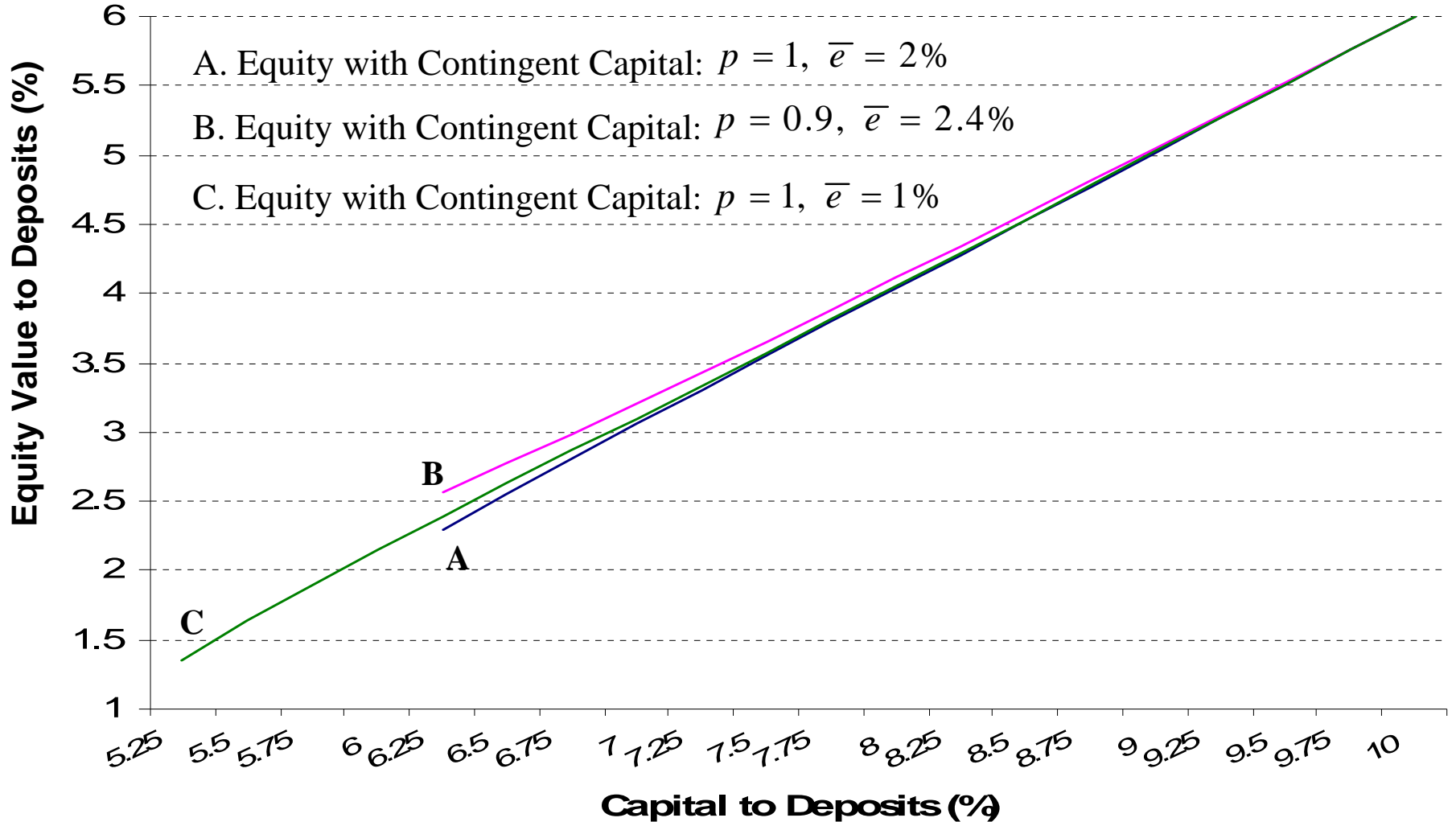*Capital to Deposits (%)* (x-axis)

38

Figure 8

New Issue Par Yields on Fixed-Coupon Contingent Capital (c)
Effect of a Dual Price Trigger
(Five-Year Maturity, Initial Value = 4% of Deposits)

A. Default-Free Five-Year Coupon Bond
B. Contingent Capital: $p = 1$, $\bar{e} = 2\%$
C. Dual Price Trigger Contingent Capital: $p = 1$, $\bar{e} = 2\%$
D. Subordinated Debt
E. Contingent Capital: $p = 0.9$, $\bar{e} = 2.4\%$
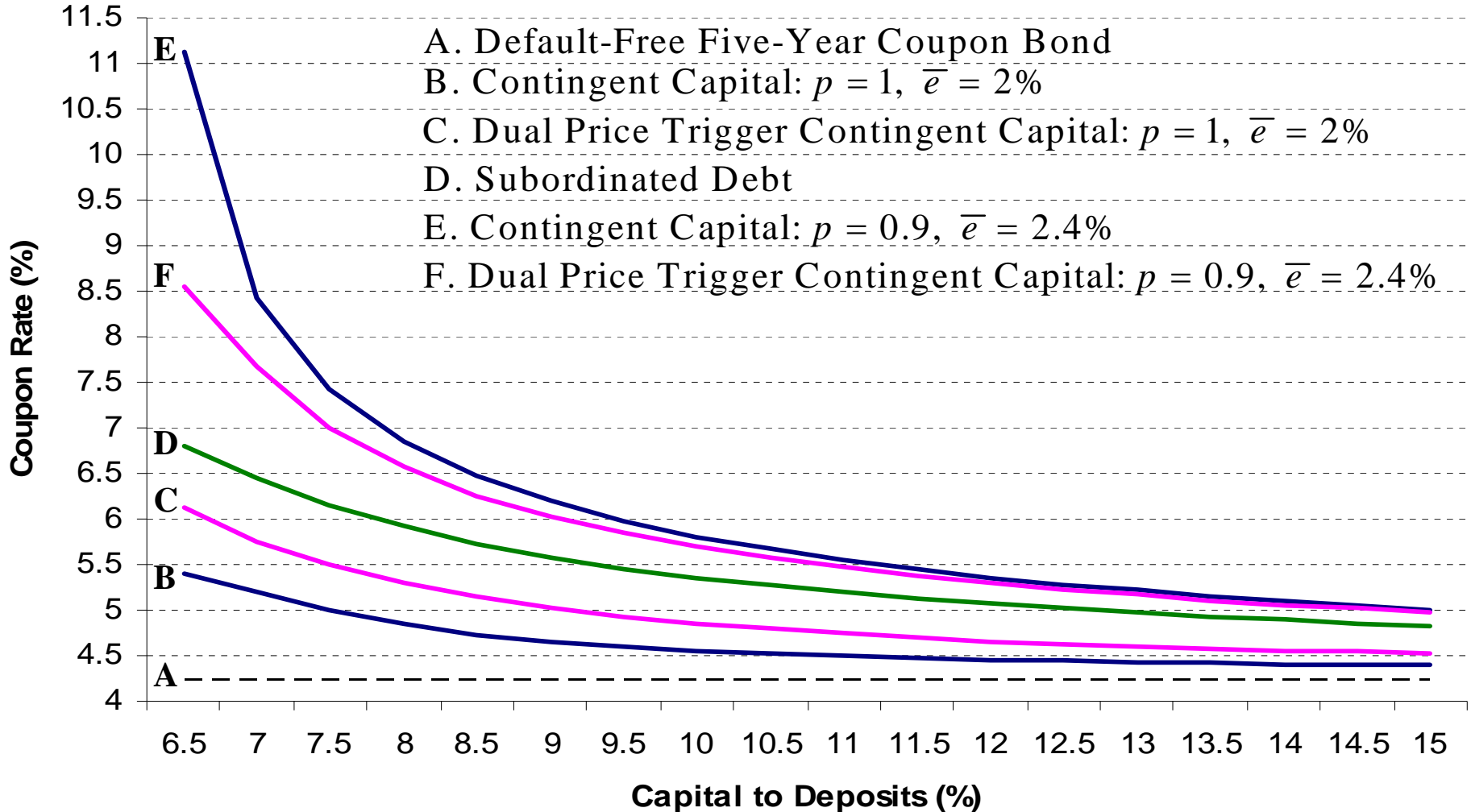F. Dual Price Trigger Contingent Capital: $p = 0.9$, $\bar{e} = 2.4\%$

Figure 9

**Change in the Value of Shareholders' Equity
For a 25% Increase in Frequency of Jumps (λ)**

(Five-Year Maturity CC and Sub Debt Having Initial Value = 4% of Deposits)

A. Contingent Capital: $p = 1$, $\overline{e} = 2\%$
B. Contingent Capital: $p = 0.9$, $\overline{e} = 2.4\%$
C. Contingent Capital: $p = 1$, $\overline{e} = 1\%$
D. Subordinated Debt

$\partial E/(\partial \lambda / \lambda)$ (%)
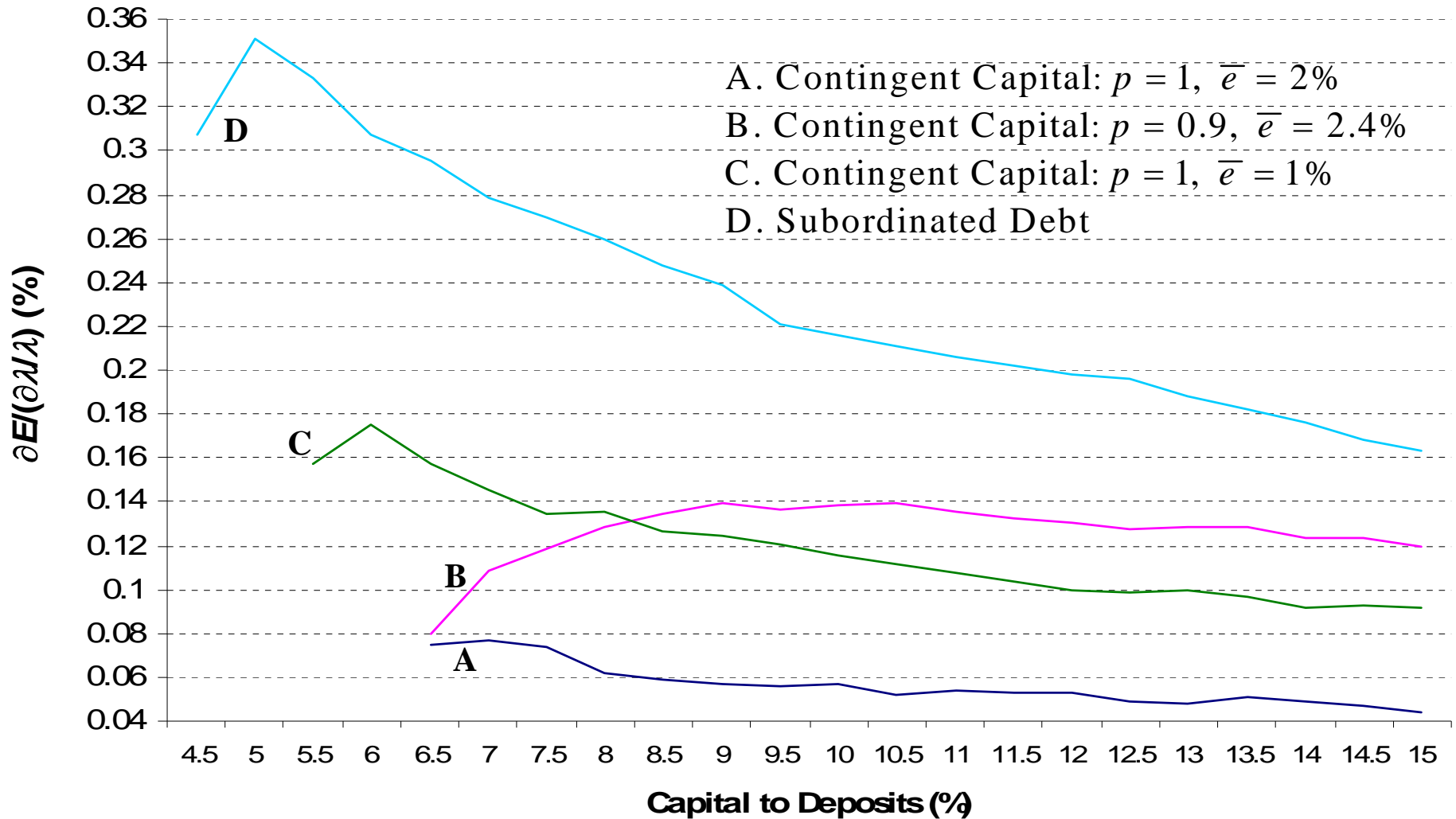
Capital to Deposits (%)

40

Figure 10

**Change in the Value of Shareholders' Equity
For a 25% Increase in the Volatility of Jumps (σ_y)**

(Five-Year Maturity CC and Sub Debt Having Initial Value = 4% of Deposits)

A. Contingent Capital: $p = 1$, $\overline{e} = 2\%$
B. Contingent Capital: $p = 0.9$, $\overline{e} = 2.4\%$
C. Contingent Capital: $p = 1$, $\overline{e} = 1\%$
D. Subordinated Debt

y-axis: $\partial E/(\partial \sigma_y / \sigma_y)$ (%)
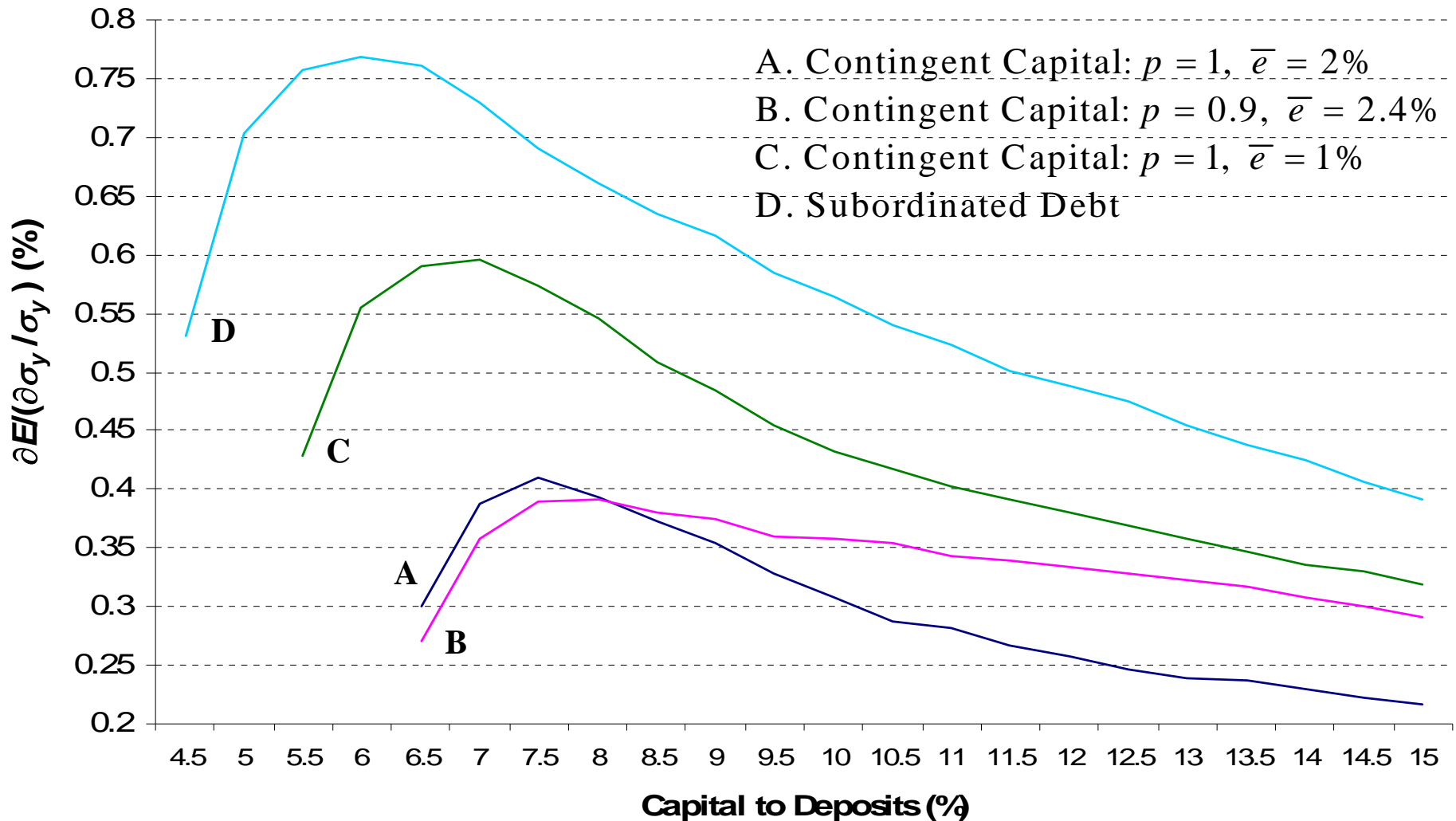
x-axis: Capital to Deposits (%)

41

Figure 11

**Change in the Value of Shareholders' Equity
For a 25% Decline in the Mean Jump Size (μ_y)**

(Five-Year Maturity CC and Sub Debt Having Initial Value = 4% of Deposits)

A. Contingent Capital: $p = 1$, $\overline{e} = 2\%$
B. Contingent Capital: $p = 0.9$, $\overline{e} = 2.4\%$
C. Contingent Capital: $p = 1$, $\overline{e} = 1\%$
D. Subordinated Debt

Y-axis: $\partial E/(\partial \mu_y / \mu_y)$ (%)
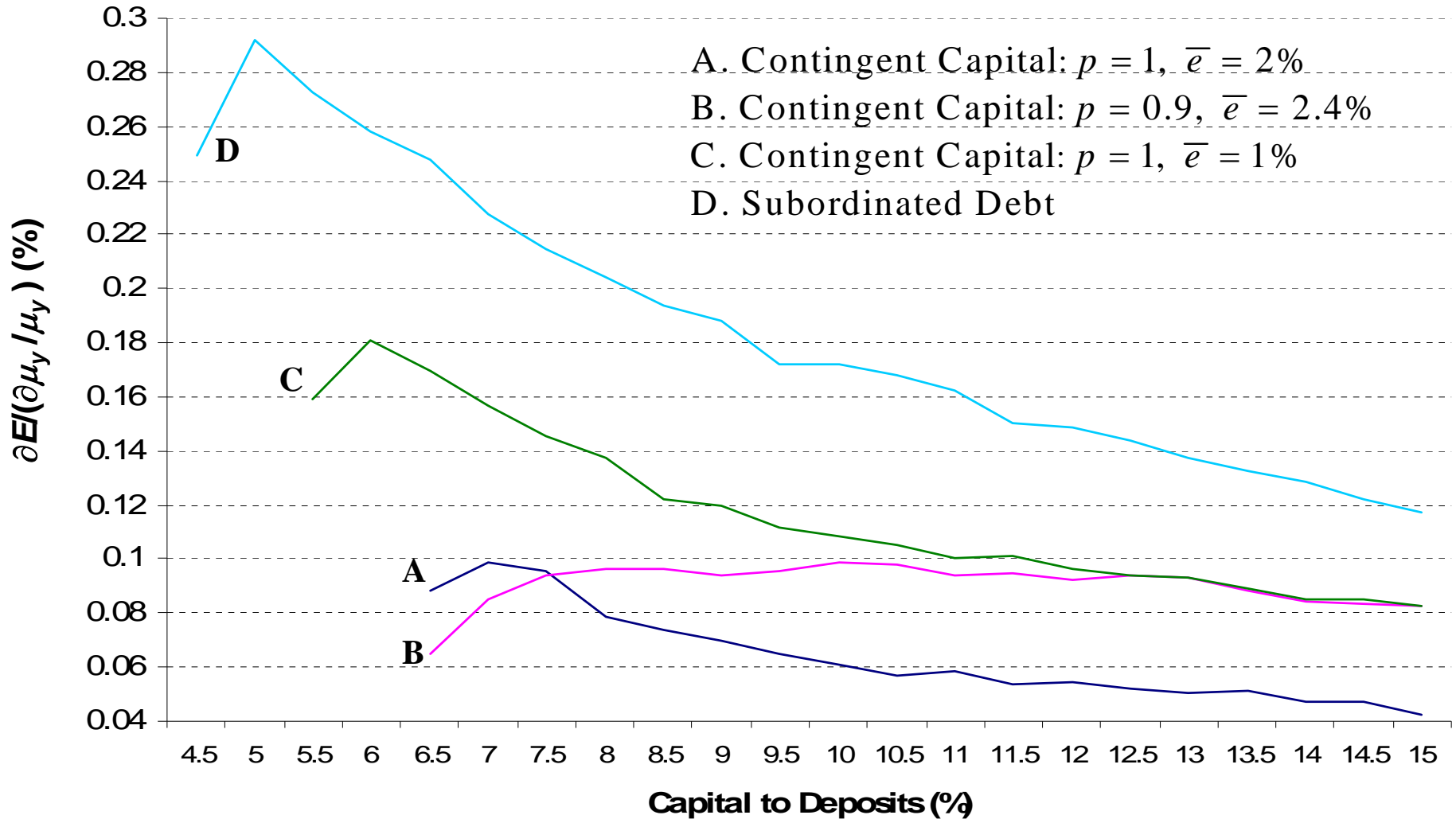
X-axis: Capital to Deposits (%)

42

Figure 12

**Change in the Value of Shareholders' Equity
For a 25% Increase in Diffusion Volatility (σ)**

(Five-Year Maturity CC and Sub Debt Having Initial Value = 4% of Deposits)

A. Contingent Capital: $p = 1$, $\overline{e} = 2\%$
B. Contingent Capital: $p = 0.9$, $\overline{e} = 2.4\%$
C. Contingent Capital: $p = 1$, $\overline{e} = 1\%$
D. Subordinated Debt

Capital to Deposits (%)

43