Differences in Tranching Methods: Some Results and Implications
Agenda

1. Preliminaries: defining tranches and tranching approaches
2. Why some tranches get skipped?
3. Do thicker tranches have lower LGD?
4. Are thicker tranches always safer?
5. Is the expected LGD on the senior tranche bounded?
6. PD and LGD of the senior tranche under EL-based tranching
7. Summary
1 Preliminaries: Defining Tranches and Tranching Approaches
Preliminaries: Defining Tranches

- An attachment point, $A$, and a detachment point, $D$, define a tranche.
- No losses to a tranche if the collateral loss is less than the attachment point.
- Tranche is wiped out if the collateral loss exceeds the detachment point.
- Tranche loss rate as a function of collateral losses is shown on the picture.
Preliminaries: Two Popular Tranching Approaches


2. Expected Loss (EL) based tranching approach: Moody’s
Preliminaries: PD based tranching approach

- A tranche is eligible for a specific rating if the tranche PD is less than a predefined value, $PD_R$.

  \[
  \text{Tranche } PD = P(L > A) = \int_A^1 f_L(L) \cdot dL \leq PD_R
  \]

- Where,
  \begin{align*}
  A &\equiv \text{tranche attachment point} \\
  L &\equiv \text{loss rate on the portfolio} \\
  f_L(\cdot) &\equiv \text{pdf of the collateral loss rate}
  \end{align*}

PD-based CE is equivalent to VaR with $\alpha = PD_R$ (idealized default rate).
Preliminaries: EL based tranching approach

\[ \text{Tranche Loss} = \min \left[ 1, \max \left( \frac{L - A}{D - A}, 0 \right) \right] \]

\[ \text{Tranche EL} = \int_{0}^{1} \text{Tranche Loss} \cdot f_L(L) \, dL \]

\[ = \int_{0}^{1} \min \left[ 1, \max \left( \frac{L - A}{D - A}, 0 \right) \right] \cdot f_L(L) \, dL \]
Preliminaries: EL based tranching approach

\[ CE_{Sr} = \min \left\{ A \left[ EL_{Sr} \geq \int_{0}^{1} \min \left[ 1, \max \left( \frac{L - A}{1 - A}, 0 \right) \right] \cdot f_L(L) \, dL \right] \right\} \]

\[ CE_{SrSub} = \min \left\{ B \left[ EL_{SrSub} \geq \int_{0}^{1} \min \left[ 1, \max \left( \frac{L - B}{Sr_{CE} - B}, 0 \right) \right] \cdot f_L(L) \, dL \right] \right\} \]
Why Some Tranches Get Skipped?
Why some tranches get skipped?

A tranche is always skipped when its idealized EL is smaller than the PD of the tranche above it.

\[
Tranche\ EL_{A,D} = \int_{A}^{D} \left( \frac{L-A}{D-A} \right) \cdot f_L(L) \, dL + \int_{D}^{1} 1 \cdot f_L(L) \, dL
\]

\[
= \int_{A}^{D} \left( \frac{L-A}{D-A} \right) \cdot f_L(L) \, dL + \left[1 - F(D)\right]
\]
3. Do Thicker Tranches Have Lower LGD?
Do thicker tranches have lower LGD?

\[
LGD_{A,D} = \frac{\int_A^D \left( \frac{L-A}{D-A} \right) \cdot f_L(L) \, dL + \int_D^1 f_L(L) \, dL}{\int_A^D f_L(L) \, dL + \int_D^1 f_L(L) \, dL}
\]

\[
LGD_{A^+,D} = \frac{\int_A^{D+} \left( \frac{L-A^+}{D-A^+} \right) \cdot f_L(L) \, dL + \int_D^1 f_L(L) \, dL}{\int_A^{D+} f_L(L) \, dL + \int_D^1 f_L(L) \, dL}
\]

Making a tranche thicker lowers the LGD of the tranche.
Are Thicker Tranches Always Safer?
Are thicker tranches always safer?
Are thicker tranches always safer?

\[
\text{Tranche } EL_{A^+,D} = \left[ \frac{1}{D-A^+} \right] \left\{ \int_0^A 0 \cdot f_L(L) dL + \int_0^{A^+} 0 \cdot f_L(L) dL + \int_{A^+}^D (L-A^+) \cdot f_L(L) dL + \int_D^1 (D-A^+) \cdot f_L(L) dL \right\}
\]

\[
= \int_{A^+}^D \left( \frac{L-A^+}{D-A^+} \right) f_L(L) dL + [1-F(D)]
\]

\[
\text{Tranche } EL_{A,D} = \left[ \frac{1}{D-A} \right] \left\{ \int_0^A 0 \cdot f_L(L) dL + \int_0^{A^+} (L-A) \cdot f_L(L) dL + \int_{A^+}^D (L-A) \cdot f_L(L) dL + \int_D^1 (D-A) \cdot f_L(L) dL \right\}
\]

\[
= \int_{A^+}^D \left( \frac{L-A}{D-A} \right) f_L(L) dL + \int_{A^+}^{A^+} \left( \frac{L-A}{D-A} \right) f_L(L) dL + [1-F(D)]
\]
Are thicker tranches always safer?

\[
\int_{A^+}^{D} \left( \frac{L-A}{D-A} \right) f_L(L) dL \geq \int_{A^+}^{D} \left( \frac{L-A^+}{D-A^+} \right) f_L(L) dL
\]

\[
\left( \frac{L-A}{D-A} \right) \geq \left( \frac{L-A^+}{D-A^+} \right) \text{ for } L \in [A^+, D]
\]

\[
\left( \frac{L-A}{D-A} \right) = 1 - \left( \frac{D-L}{D-A} \right)
\]

Lowering attachment point of a tranche *increases* EL
Is the Expected LGD on the Senior Tranche Bounded?
Is the expected LGD of the senior tranche bounded?

Let $CE_{SR} = A$

$$LGD_{SR} = \frac{1}{A} \int_{L}^{1} \left( \frac{L - A}{1 - A} \right) \cdot f_{L}(L) \, dL$$

$$\leq \int_{A}^{1} \left( \frac{L - A}{1 - A} \right) \cdot f_{L}(L) \, dL = \int_{A}^{1} \left( \frac{L - A}{1 - A} \right) \cdot dL$$

$$\leq \int_{A}^{1} f_{L}(L) \, dL = \int_{A}^{1} dL$$
Is the expected LGD of the senior tranche bounded?

\[
\begin{align*}
\text{LGD} &= \left(1 - A\right) \cdot \int_A^1 \left(1 - A\right) \cdot dL \\
&= \frac{1}{\left(1 - A\right)^2} \cdot \left[\int_A^1 L \cdot dL - \int_A^1 A \cdot dL\right] \\
&= \frac{1}{\left(1 - A\right)^2} \cdot \left[\left(\frac{1 - A^2}{2}\right) - \left(A \cdot (1 - A)\right)\right] \\
&= \frac{1}{2 \cdot \left(1 - A\right)^2} \cdot \left[(1 - A^2) - 2A + 2A^2\right] \\
&= \frac{(1 - A)^2}{2 \cdot (1 - A)^2} = 0.5 \\
\Rightarrow \text{LGD}_{SR} &\leq 0.5
\end{align*}
\]
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PD and LGD of the Senior Tranche under EL-based Tranching
## PD and LGD of the Senior Tranche under EL-based Tranching

### Table 1

<table>
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<th>PD (bp)</th>
<th>Rho</th>
<th>Sr Tranche attachment point (%)</th>
<th>Sr Tranche PD (bp)</th>
<th>Sr Tranche Expected LGD (%)</th>
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<td>28</td>
<td>18.18</td>
</tr>
</tbody>
</table>

Lower Tranche LGD leads to higher tolerance for default risk and hence lower Credit Enhancement.
Summary
Summary

- Under EL-based tranching approach it is often the case that some tranches will be skipped. Under PD-based tranching approach, by construction, no tranche will be skipped.

- Making a tranche thicker lowers its LGD.

- Lowering the attachment point of a tranche (while keeping the attachment point fixed) increases its EL regardless of the probability distribution of the collateral loss.

- The highest expected LGD possible for the senior tranche is 50% 0.5 given some realistic assumptions on the collateral loss distribution. Practically, the expected LGD of the senior tranche is likely to be much smaller than 50%.

- The relative conservatism of EL-based approach compared to PD-based tranching approach depends on a number of factors, including EL and PD targets.

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